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Abstract

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INVARIANT CHARGE AND REGGE ASYMPTOTICS

Attempts to obtain information about the asymptotic ultraviolet behavior of Green' s functions in quantum field theory, proceeding from the usual renormalized perturbation theory, found their most consistent embodiment within the framework of the renormalization-group (r. g.) method ([1]). It was also established that combining the r. g. method with analyticity properties makes it possible to avoid difficulties associated with the appearance of the well-known logarithmic poles, and to obtain ([2]) a certain model asymptotic behavior of the invariant charge (i. c.), containing terms nonanalytic in the charge and leading to a finite value of the i. c. at infinity. Thus, in quantum electrodynamics, where the i. c. coincides with the photon Green' s function, it was obtained that

$$\lim_{k^2 \to -\infty} e^2 d(k^2) \equiv e^2 Z_3^{-1} = 3\pi. \tag{1}$$

The numerical value (1) is the result of calculations in the lowest order (summation of terms of the form $(e^2 \ln k)^n$ in the spectral function of the Källén-Lehmann representation). It changes when subsequent terms (of the form $e^2(e^2 \ln k)^m$) are taken into account. However, the property of finiteness of the quantity $e^2 Z_3^{-1}$ is preserved in any finite order. We shall call it the property of finiteness of the asymptotic value of the invariant charge. It was noted ([2]) that this property is not a privilege of quantum electrodynamics and occurs, in particular, also in the theory of meson-nucleon interactions. Moreover, one may assert that in the two-charge meson theory ([3]) the combination of the r. g. method with analyticity properties leads to finite values of the asymptotic i. c.' s. It may now be regarded as established that the low-energy pion-pion interaction is characterized by attraction, i.e., that the renormalized Lagrangian of the pion-pion interaction is positive. This means, in particular (see ([4])), that the finite value of the pion-pion i. c. arises due to terms nonanalytic in the pion-nucleon interaction.

Various asymptotics of higher Green' s functions, including the physical asymptotics of scattering amplitudes, can also be investigated by the r. g. method. Such an attempt was first undertaken in ([5]). There, however, considerations of analyticity were not taken into account. It can be shown that the formulas of ([5]) incorrectly reproduce terms containing squares and higher powers of logarithms.

The error consists in the fact that the r. g. formulas were used in a region where the scattering amplitude f is a complex quantity. At the same time, a scale transformation from the point where f is real (the physical normalization point) to a point where f is complex cannot be described by the operation of multiplication by a real renormalization constant. This fact was not taken into account in ([5]). It can be reflected in the following rule for combining the properties of renormalization invariance and analyticity:

The object of the renormalization-group equations must be the spectral densities of the analytic representations of the Green functions and scattering amplitudes, which are, by definition, real functions.

Let us turn directly to the asymptotic behavior of the elastic scattering amplitude. Suppose that for it there exists the Mandelstam double spectral representation ([6])

$$\begin{aligned} f(s,u,t) = {} & \frac{1}{\pi}\int \frac{A_1(s')\,ds'}{s'-s} + \frac{1}{\pi}\int \frac{A_2(u')\,du'}{u'-u} + \frac{1}{\pi}\int \frac{A_3(t')\,dt'}{t'-t} \\ & + \frac{1}{\pi^2}\int \frac{\rho_{12}(s',u')\,ds'\,du'}{(s'-s)(u'-u)} + \frac{1}{\pi^2}\int \frac{\rho_{13}(s',t')\,ds'\,dt'}{(s'-s)(t'-t)} + \frac{1}{\pi^2}\int \frac{\rho_{23}(u',t')\,du'\,dt'}{(u'-u)(t'-t)}. \end{aligned} \tag{2}$$

The rule formulated means now that the r.g. equations must be written for the one-dimensional spectral densities A_i and the double spectral functions ρ_{ik}.

For illustration we consider the theory of scattering of neutral mesons with Lagrangian

$$L(x) = \frac{\pi}{12} h\varphi^4(x). \tag{3}$$

In this theory the invariant charge has the form

$$\rho(s) = \frac{h}{1-3hF(s)}, \tag{4}$$

where the function

$$F(s) = \frac{s}{\pi}\int_{4m^2}^{\infty} \sqrt{\frac{z-4m^2}{z}}\,\frac{dz}{z(z-s)} \underset{s\to\infty}{\longrightarrow} -\frac{1}{\pi}\ln(-s) \tag{5}$$

describes the second-order Feynman "fish" diagram.

Calculating the one-dimensional spectral functions $A_1 = A_2 = A_3$ by means of the r.g., we obtain the following contribution to the scattering amplitude:

$$f(s,u,t) \simeq h^2\{I(s)+I(u)+I(t)\}; \tag{6}$$

$$I(s)=\frac{1}{\pi}\int_{4m^2}^{\infty}\frac{\sqrt{\dfrac{z-4m^2}{z}}}{(1-3hF(z))^2+9h^2\dfrac{z-4m^2}{z}}\,\frac{dz}{z-s}. \tag{7}$$

Expression (6) correctly conveys the asymptotic behavior of the leading logarithmic diagrams in all three invariant arguments and, in particular, all iterations of the "fish." At the same time, the "fish" diagrams from different arguments do not interfere with one another. Expression (6), however, does not take account of the double spectral functions and therefore turns out to be insufficient for studying asymptotic diffraction scattering in the region $s \simeq -u \gg m^2,\ t \simeq m^2$.

In work ([7]), to study diffraction asymptotics it was proposed to use the r.g. equations.

The Regge behavior

$$f(s,t) \simeq \beta(t)s^{\alpha(t)} \tag{8}$$

for the theory with Lagrangian (3) was obtained there by reasoning which, as can be shown*, is equivalent to the summation of a certain class of ladder diagrams. However, it can be shown that taking account of the remaining leading logarithmic diagrams destroys result (8).

* I am grateful to the authors of work ([7]) for explaining this important circumstance.

We shall show below that the Regge asymptotic behavior (8) can be obtained by the R.G. method in an internally consistent way, on the basis of the property of finiteness of the asymptotic I.C.

It is clear that, in order to obtain expression (8), it is necessary to consider the properties of the double spectral functions $\rho(s,t)$, for only they give contributions to f that depend simultaneously on the two arguments s and t. In the R.G. equations one should substitute the results of calculating the spectral functions in the fourth and higher orders of perturbation theory. Such a consideration, which is of great interest for the real symmetric theory of charged pions, lies beyond the scope of the present communication because of the cumbersome nature of the calculations. Since we have in any case restricted ourselves to the neutral model, we shall give only the scheme of this argument.

Let us consider the first logarithmic terms of perturbation theory

$$f_{\rm t.v}(s,t;h) = h - \frac{h^2}{\pi}[\ln s + \ln(-s)] + 3\frac{h^3}{\pi^2}[\ln^2 s + \ln^2(-s)] + \frac{h^4}{\pi^3}a(t)[\ln s + \ln(-s)] + \dots . \tag{9}$$

Here the terms h^2 and h^3 correspond to the iterations of the "fish" diagrams and the "frame" diagrams and are expansions of the one-dimensional integrals (7). The last term of order h^4 is associated with the "barrel" diagram. In the corresponding expression for $\rho(s,t)$, the role of the term h is played by the contribution from the "barrel," while the analogue of the term h^4 arises from more complicated diagrams.

Carrying out the integration of the group equation, we obtain

$$\ln\frac{f(s,t;h)}{f(s_0,t;h)} = \int_{s_0}^{s}\frac{dz}{z}\left[\frac{\partial}{\partial\xi}\ln f_{\rm t.v}\left(\xi, \frac{t}{z}, \frac{m^2}{z}, \rho(z)\right)\right]_{\xi=s} =$$

$$= \int_{s_0}^{s}\frac{dz}{z}\left\{-2\frac{\rho(z)}{\pi} + 2\frac{\rho^3(z)}{\pi^3}a(t)\right\}. \tag{10}$$

It is clear that, if formula (4) is substituted into (10), we shall not obtain Regge behavior. The only possible form of the asymptotic behavior of the invariant charge $\rho(z)$ leading to (8) is, evidently,

$$\lim_{z\to\infty}\rho(z) = \pi H, \qquad 0 < |H| < \infty. \tag{11}$$

Substituting (11) into (10), we arrive at formula (8), with

$$\alpha(t) = -2H + 2H^3 a(t). \tag{12}$$

Formula (12) still does not give us the possibility of discussing the properties of the Regge trajectory arising from perturbation theory. As was said, for this it is necessary to carry out calculations with the double spectral function.

Let us emphasize, however, the main consequences of result (12), which carry over directly to a more complete consideration:

1. Regge asymptotics can be obtained in quantum field theory by the R.G. method, starting from perturbation theory for the spectral functions of the Mandelstam representation, under the assumption that the invariant charge tends to a finite asymptotic value πH. This possibility is closely connected with the nonanalyticity of the scattering matrix in the coupling constant (cf. (8)).
2. In the explicit expression for the trajectory $\alpha(t)$, alongside low-energy information from perturbation theory (the coefficient $a(t)$), there enters

the asymptotic value of the invariant charge H, i.e., essentially high-energy information described by a single parameter*.

3. The numerical value of H can be determined from the condition $\alpha(0) = 1$ (the Pomeranchuk regime) and turns out to be a quantity of order unity.

4. The expression for the trajectory is a series in increasing powers of H, and the terms of this series apparently do not decrease. Therefore, hopes of obtaining any reasonable expression for $\alpha(t)$ may be based on an analysis of the character of the threshold and asymptotic singularities of coefficients of the type $a(t)$, as a result of which the series may perhaps be summed by the renormalization-group method.

5. The universality of the vacuum trajectory $\alpha(t)$ for all elastic processes $a + b \to a + b$ should find its reflection in relations between the asymptotic invariant charges of different interactions.

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* This conclusion contradicts the result of work (9), in which (8) was obtained by summing ladder diagrams with the aid of the Bethe–Salpeter equation.

Note: Figure translations are in progress. See original paper for figures.