SOME QUESTIONS IN THE THEORY OF NAVIGATION

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Abstract

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SOME QUESTIONS IN THE THEORY OF NAVIGATION

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At the present time, no sufficiently general definition has been given of the concept of navigational information, and the whole variety of existing navigational methods (radio-navigational, compass, astronomical, inertial, acoustic, etc.) is not reducible to any single physical principle. This circumstance leads to certain difficulties, both methodological and scientific-technical. It seems possible to outline a general approach to the analysis of various navigational methods, relying on the modern theory of space, time, and gravitation (1).

First of all, let us clarify the content of the concept of navigational information. Navigational information is one of the kinds of information about events in the material world around us. One of the characteristics of material bodies is their mutual position in space, in time or, in the general case, in the space-time manifold. It is precisely this characteristic that may constitute the content of the concept of navigational information. Therefore, navigational information should be understood as **information about the relative coordinates of bodies in a four-dimensional space-time manifold**. The relative coordinates of bodies are quantities characterizing the mutual position of the centers of mass of the bodies, the mutual orientation of axes connected with the bodies, as well as derivatives of various orders of these quantities.

Such an interpretation assumes that, in the mathematical characterization of navigational information, one cannot confine oneself only to determining the quantity of information; computations are also necessary that follow from the qualitative specificity of this kind of information. This agrees with the general propositions of information theory considered in (2).

The centers of mass of bodies may be regarded as points in a four-dimensional manifold, whose mutual position is determined by a space-time interval. To find this interval it is necessary to compare the events at the points under consideration. Such a comparison can be performed with the aid of certain material bodies or processes capable of moving between these points and having the significance of signals. Thus, various bodies and physical processes moving between the points being compared may serve as navigational signals, provided only that the velocity and trajectory of their motion are determined. Let us note that, up to the present, only electromagnetic, gravitational, and acoustic disturbances have been used as navigational signals.

An objective fact accompanying the displacement of a signal is the change in the coordinates of the signal. However, in order to detect this change, it is necessary to compare the coordinates characterizing some two positions of the signal. It should therefore be considered that navigational information is formed at the place and at the time where and when, by means of the signal, a comparison of events at the points of interest to us has been carried out. The place of formation of navigational information is the comparing

device (navigation indicator, antenna system, comparison circuit, etc.).

The medium in which the navigation signal moves influences the process of producing navigation information. The distribution of matter in space determines the metric of the space and, consequently, the trajectory of the signal. Knowledge of the metric of space thus gives navigation information quantitative definiteness.

It is useful to introduce the concept of a navigation space, understanding by this the space in which the signal moves and in which navigation problems are solved. When modern navigation aids are used, navigation spaces do not always possess a Euclidean metric. Thus, when radio-navigation systems operating on long waves diffracting around the Earth are used, the two-dimensional navigation space will have the form of one or another region of the geoid, and the signal trajectories will be segments of geodesics on the surface of the geoid. In another case, when radio-navigation systems operating on short or ultrashort waves undergoing refraction in the atmosphere are used, the trajectories of navigation signals turn out to be curved, and the navigation space will be characterized by a non-Euclidean metric. Speaking of extraterrestrial navigation, it must be borne in mind that the trajectories of space objects have a distinctly curvilinear form, and the trajectories of navigation signals in outer space will likewise not always be rectilinear, since space itself is curved by gravitational fields.

It is necessary to emphasize that usually, in developing navigation methods, one strives to solve problems in Euclidean space of two or three dimensions. In so doing, the effects caused by the deviation of the metric of the navigation space from the Euclidean one are accounted for by introducing corresponding corrections into the navigation calculations (corrections for the sphericity or spheroidicity of the Earth, corrections for refraction and delay in the ionosphere, etc.). From a methodological point of view, such an approach is not adequate to the problem posed.

It seems advisable not to ascribe to navigation space an obligatory Euclidean metric. It is more correct to regard navigation space, in the general case, as a Riemannian space, and therefore the deviation of the metric of this space from the Euclidean one is its nominal property. **Methods for solving navigation problems must depend on the metric properties of the navigation space**, and navigation algorithms in non-Euclidean spaces must be a generalization of the algorithms of ordinary Euclidean navigation.

The development of navigation methodology must be based on an analysis of the metric of the given navigation space and of the properties of geodesic lines in it. Thus, for example, an exact methodology of navigation on the surface of the Earth should be constructed on the basis of an analysis of the properties of geodesics on the surface of the ellipsoid to which the geoid is likened. Navigation algorithms in this case must be based on formulas giving the functional relation between the lengths of geodesics and orthogonal surface coordinates and geodesic azimuths.

In the process of navigation determinations, as a result of comparing events at points connected by a signal, the space-time relation of these points is established. Its analytical expression is the four-dimensional interval ([1]):

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta,$$

where x^γ are coordinates ($\alpha,\ \beta = 0,\ 1,\ 2,\ 3$), and $g_{\alpha\beta}$ are the components of the fundamental metric tensor.

In accordance with this, the primary navigation formula may be written in the following most general form:

$$\int_{p_1}^{p_2} \left[g_{00} k^2 \left(\frac{\partial N}{\partial p} \right)^2 + 2 g_{0i} k \frac{\partial N}{\partial p} \frac{\partial x^i}{\partial p} + g_{ik} \frac{\partial x^i}{\partial p} \frac{\partial x^k}{\partial p} \right]^{1/2} dp - c\tau = 0. \quad (1)$$

Here p is a parameter by means of which x^ν and $g_{\alpha\beta}$ can be specified; N is a navigation parameter admitting direct measurement by the use of technical means; k is the scale coefficient of the measurements, in turn expressed through the time coordinates t_i in the form: $k = (t_2 - t_1)(N_2 - N_1)^{-1}$; c is the speed of light; τ is the interval of proper time in the signal system.

In this case, for Euclidean space-time geometry, taking into account that then $g_{00} = c^2$, $g_{0i} = 0$, and $g_{ik} = -\delta_{ik}$ $(i, k = 1, 2, 3)$, we have:

$$(\delta_{ik} \Delta x^i \Delta x^k)^{1/2} - c \left[(k\Delta N)^2 - \tau^2 \right]^{1/2} = 0. \quad (2)$$

Formulas (1)–(2) ultimately give the relation between increments of the geometric coordinates and the increment of time corresponding to the passage of a signal between the points being compared. From these dependences follows the possibility of posing two entirely equivalent types of problems: problems of finding spatial coordinates from known time coordinates, and problems of finding time coordinates from known spatial coordinates. It therefore seems appropriate to generalize the concept of a navigation determination, including in it, alongside a navigation-spatial determination, also a navigation-temporal determination. Navigation-spatial determinations are the traditional problem of navigation of moving objects (ships, aircraft, etc.). Navigation-temporal determinations have acquired practical significance only in recent years in the form of the problem of synchronizing clocks at remote points, which is connected with the operation of a unified time network used, for example, for time reference of observations of artificial Earth satellites and space rockets.

It is important to emphasize that both these problems are two sides of a single physical process associated with the production of navigation information.

At the basis of the navigation problems of moving objects lies the determination of spatial coordinates. Therefore navigation-spatial methods are based on the measurement of time intervals. Different navigation means differ from one another in the technical methods of measuring these time intervals, and also in the procedures for forming, on the basis of these measurements, various secondary navigation parameters.

In modern navigation devices, to measure time intervals use is made either of clocks (mechanical, electronic), or of processes whose parameters are functionally related to the passage of time. In the latter case time is modeled on a certain scale. The coefficient k in expression (1) is essentially the scale coefficient of the modeling.

Thus, when radio waves are used as navigation signals, their phase and frequency are employed for modeling time. In the case of phase measurements ($N = \psi$) the scale coefficient k is equal to $k_\psi = (4\pi f)^{-1}$, where f is the frequency. In the case of frequency measurements ($N = F$), $k_F = (8\Delta f F_{\mathrm{M}})^{-1}$, where Δf is the frequency deviation and F_{M} is the modulation frequency. Let us add that for time measurements ($N = T$), used in pulse devices, the scale coefficient $k_T = 1$. Obviously, from the fundamental-methodological point of view, time, phase, and frequency techniques are equivalent.

Besides the measurement of absolute values of time intervals, difference measurements are realizable, giving the difference of time intervals along two signal paths. A particular but very widespread case of these is measurements with the reduction of the difference of time intervals to zero.

The geometric equivalent of a time interval is the segment of a geodesic connecting the points being compared, and the geometric equivalent of the difference of time intervals is the difference of geodesic segments. Therefore, navigation-spatial information in its primary form is expressed in the form of geodesic distances and their differences (including zero differences). Successive measurements in time of first and second differences of geodesic distances, performed along some direction, give the values of velocities and accelerations of the navigation object.

The common feature of the various methods used to measure distances and differences of distances—radio-engineering methods (with phase, frequency, time, and correlation meters), acoustic methods (with transmission of the reference origin by radio signals), and optical methods (using coherent sources)—is that, by means of different technical procedures, one and the same physical quantities are measured: the absolute values of time intervals and the differences of these intervals. Let us note that if it were possible to generate, modulate, and selectively receive gravitational waves of the required power, gravitational rangefinders could appear.

The common feature of the various methods used for sighting directions (and, on this basis, for measuring angles), such as radio-engineering methods (radio direction finders, radio beacons, radio-astronomical devices), optical methods (vertical circles, sextants, orienters), acoustic methods (locators), magnetic methods (compasses), gravitational methods (plumb lines), etc., is that their use is based on finding the extremal (zero) value of the difference of time intervals along nearby parallel paths.

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CITED LITERATURE

[1] V. A. Fok, *Theory of Space, Time, and Gravitation*, Moscow, 1961. [2] A. N. Kolmogorov, "Theory of Information Transmission," in: *Collection. Session of the Academy of Sciences of the USSR on Scientific Problems of Production Automation, 15-20 X 1956, Plenary Sessions*, Publishing House of the Academy of Sciences of the USSR, 1957.

Note: Figure translations are in progress. See original paper for figures.