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Abstract

Full Text

GEOPHYSICS

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ON THE DETERMINATION OF THE INITIAL FIELDS OF PRESSURE AND WIND FROM THE DISTRIBUTIONS OF TEMPERATURE AND VERTICAL AIR MOTIONS

The problem of determining atmospheric-pressure fields from given temperature fields can be solved only by invoking the dynamics of atmospheric motions. Indeed, if we know the distribution of temperatures in the Earth' s atmosphere as functions of latitude, longitude of place, and height above sea level, then, owing to the quasistatic nature of atmospheric processes, we can, from the equation of statics, by carrying out a simple quadrature, find the pressure at all levels, but only up to an arbitrary function of latitude and longitude (for example, up to the pressure at sea level). A method for determining this arbitrary function by using the equations of dynamics was given by us ([1,2]) for the stationary case —in the problem of determining the centers of action of the atmosphere. The question arises whether, in the general nonstationary case as well, it is possible to construct a diagnostic equation (an equation not containing differentiation with respect to time) that would make it possible to close the problem of determining pressure. We shall show that this can readily be done, while remaining within the framework of quasigeostrophy, if, in addition to temperature, one also includes the vertical velocities of the air in the consideration.

For simplicity we shall start from the system of equations adopted by us in paper ([3]):

$$\frac{\partial \Delta\psi}{\partial t} + \frac{1}{a_0^2 \sin\theta}(\psi, \Delta\psi) + 2\omega\frac{\partial\psi}{\partial\lambda} - \frac{4\omega^2}{g\sin\theta}\left(\psi\cos\theta,\ \frac{\partial\psi}{\partial z}\cos\theta\right) =$$

$$= 2\omega a_0^2 \cos\theta\,\frac{1}{\tilde{\rho}}\,\frac{\partial\tilde{\rho} v_z}{\partial z}; \tag{1}$$

$$\frac{\partial^2\psi}{\partial z\,\partial t} + \frac{1}{a_0^2\cos\theta\sin\theta}\left(\psi,\ \frac{\partial\psi}{\partial z}\cos\theta\right) + \frac{g(\gamma_a - \gamma)}{2\omega\cos\theta T_1}\,v_z = 0; \tag{2}$$

$$\frac{\partial\psi}{\partial z} = \frac{g}{2\omega\cos\theta T_1}\,T' \tag{3}$$

(the vorticity-transfer equation, the heat-inflow equation, and the static equation). Here ψ is the stream function; v_z is the vertical component of velocity; T' is the deviation of the temperature T from its standard value $\tilde{T}(z)$; t is time; θ and λ are the complement of latitude and longitude of the place, respectively; z is height above sea level; a_0 is the radius of the Earth; $\tilde{\rho}(z)$ is the standard density; ω is the angular velocity of rotation of the Earth; T_1 is the mean temperature of the atmosphere; g is the acceleration due to gravity; γ_a is the adiabatic temperature gradient; γ is the vertical temperature gradient;

$$(A, B) = \frac{\partial A}{\partial \theta}\frac{\partial B}{\partial \lambda} - \frac{\partial A}{\partial \lambda}\frac{\partial B}{\partial \theta}.$$

If the stream function ψ is known, then the horizontal velocities v_θ and v_λ are found from the formulas

$$v_\theta \simeq -\frac{1}{a_0 \sin\theta}\frac{\partial \psi}{\partial \lambda}, \qquad v_\lambda \simeq \frac{1}{a_0}\frac{\partial \psi}{\partial \theta}, \tag{4}$$

and the deviation p' of the pressure p from its standard value $\tilde{p}(z)$ is determined, by the condition of quasigeostrophy, from the relation

$$p' = 2\omega\tilde{\rho}\cos\theta\,\psi. \tag{5}$$

Can one construct a diagnostic equation for ψ? We eliminate the time derivatives from equations (1) and (2). To do this, we differentiate (1) with respect to z, take the Laplace operator Δ of (2), and equate the expressions for $\frac{\partial}{\partial z}\frac{\partial \Delta\psi}{\partial t}$ and $\Delta\frac{\partial^2 \psi}{\partial z\,\partial t}$.

We obtain

$$\frac{1}{a_0^2 \sin\theta}\frac{\partial}{\partial z}(\psi, \Delta\psi) + 2\omega\frac{\partial^2\psi}{\partial z\,\partial\lambda} - \frac{1}{a_0^2}\Delta\left[\frac{1}{\cos\theta\sin\theta}\left(\psi, \frac{\partial\psi}{\partial z}\cos\theta\right)\right] -$$

$$-\frac{4\omega^2}{g\sin\theta}\frac{\partial}{\partial z}\left(\psi\cos\theta, \frac{\partial\psi}{\partial z}\cos\theta\right) =$$

$$= \frac{g}{2\omega T_1}\Delta\left(\frac{\gamma_a - \gamma}{\cos\theta}v_z\right) + 2\omega a_0^2\cos\theta\frac{\partial}{\partial z}\left(\frac{1}{\tilde{\rho}}\frac{\partial\tilde{\rho}v_z}{\partial z}\right). \tag{6}$$

If ψ is known, then equation (6) can serve to determine the initial values of v_z (with boundary conditions: $\tilde{\rho}v_z = 0$ at $z = \infty$ and $v_z = 0$ at $z = 0$). Suppose now that T' and v_z are known to us. Performing in the left-hand side of (6) the differentiation with respect to z and expressing $\partial\psi/\partial z$ from (3) through T', we obtain a linear equation for determining ψ. Namely:

$$(\Delta\psi, \tau) - (\psi, \Delta\tau) + \sin\theta \cdot \Delta\left[\frac{1}{\sin\theta\cos\theta}(\psi, \tau\cos\theta)\right] +$$

$$+\frac{4a_0^2\omega^2}{g}(\psi\cos\theta, \tau\cos\theta) = \sin\theta F, \tag{7}$$

$$F = 2\omega a_0^2\frac{\partial\tau}{\partial\lambda} - \frac{2\omega T_1 a_0^2}{g}\left[2\omega a_0^2\cos\theta\frac{\partial}{\partial z}\left(\frac{1}{\tilde\rho}\frac{\partial\tilde\rho v_z}{\partial z}\right) + \frac{g}{2\omega T_1}\Delta\left(\frac{\gamma_a - \gamma}{\cos\theta}v_z\right)\right].$$

Here $\tau = T'/\cos\theta$. Equation (7) contains no differentiation of the desired function ψ with respect to z. It can be solved separately for each level z; in the solution the height z will enter parametrically.

Equation (7) takes an especially simple form if it is applied to the determination of ψ at the mean level of the atmosphere. Introducing $\xi = \tilde p/P$ (P is the standard pressure at sea level), we can write for the mean level

$$\frac{\partial}{\partial z}\left(\frac{1}{\tilde\rho}\frac{\partial\tilde\rho v_z}{\partial z}\right) = \frac{g^2}{P^2}\tilde\rho\frac{\partial^2\tilde\rho v_z}{\partial\xi^2} \simeq$$

$$\simeq \frac{g^2}{P^2}\tilde\rho\,\frac{(\tilde\rho v_z)_{\xi=0} - 2(\tilde\rho v_z)_{\xi=0.5} + (\tilde\rho v_z)_{\xi=1}}{0.25}.$$

Since $\tilde\rho v_z$ vanishes at $z = \infty$ ($\xi = 0$) and at $z = 0$ ($\xi = 1$), we may write

$$\left\{\frac{\partial}{\partial z}\left(\frac{\rho}{\tilde\rho}\frac{\partial\tilde\rho v_z}{\partial z}\right)\right\}_{\xi=0.5} \simeq -8\frac{g^2}{P^2}(\tilde\rho^2 v_z)_{\xi=0.5}. \tag{8}$$

If we now write equation (7) for the middle level ($\xi = 0.5$) and use (8), then we obtain an equation from which we can find ψ at the middle level from the distribution of τ and v_z at the same level.

Our diagnostic equation will have an even simpler form if we take $\psi = \bar\psi(\theta, z) + \tilde\psi(\theta, \lambda, z)$, where $\bar\psi$ is a known function determined from the climatological zonal values of the stream function ψ; $\tilde\psi$ is an unknown small quantity. In this case, for the known function τ one may take the representation $\tau = \bar\tau(\theta, z) + \tilde\tau(\theta, \lambda, z)$, where $\tilde\tau$ is a small quantity. If the diagnostic equation is now linearized, discarding squares and products of $\tilde\psi$ and $\tilde\tau$, then a simple equation is obtained for determining $\tilde\psi$ in terms of $\tilde\tau$ and v_z. One may take $\bar\psi = -\alpha(z)a_0^2\cos\theta$, $\bar\tau = -M(z)\cos\theta$ [$\tau = T'(\theta, \lambda, z)/\cos\theta = -M(z)\cos\theta + \tilde T(\theta, \lambda, z)/\cos\theta$, $\tilde\tau =$

$\tilde{T}(\theta, \lambda, z)/\cos\theta$; $T = T_0(z) + M(z) - M(z)\cos^2\theta + T'(\theta, \lambda, z)]$. Here $T_0(z)$ is the standard temperature at the pole, $\alpha(z)$ is the angular velocity of air motion relative to the Earth—the "circulation index" in the terminology of Rossby and Gaurwitz. The quantity $M(z)$ is the temperature difference between the equator and the pole.

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CITED LITERATURE

[1] E. N. Blinova, *DAN*, **39**, No. 7 (1943).
[2] E. N. Blinova, *DAN*, **92**, No. 3 (1953).
[3] E. N. Blinova, *DAN*, **110**, No. 6 (1956).

Note: Figure translations are in progress. See original paper for figures.

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