A SHOCK WAVE OF FRACTURE IN BRITTLE MEDIA

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CONTINUUM MECHANICS

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A SHOCK WAVE OF FRACTURE IN BRITTLE MEDIA

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In agreement with experimental data ([1]), one may conclude that, during an explosion in a massif of brittle material, near the gas chamber of the detonation products there arises a developing zone of fracture, inside which the crushed, comminuted material is subjected to high pressures, while the unfractured part of the massif experiences deformations propagating at high velocity.

It is natural to assume that the process of explosion in this medium is accompanied by the formation of a shock wave of fracture, representing a certain isolated surface, across which the principal characteristics of the motion—density, pressure, velocity—undergo a discontinuity. In the fracture wave, the jump-like transition of the material from a strong state to a fractured state is accomplished within the framework of the hypothesis of a "continuous medium." This idea was first expressed in 1957 by Kh. A. Rakhmatulin.

Fig. 1. Model of an explosion

Statement of the problem. The fracture wave arises under the action of the explosion of a charge of radius R_{00} (see Fig. 1). The characteristics of the state of the particles on different sides of the wave, taking into account the spherical symmetry of the motion, are connected by the relations

$$\rho_1(\dot{R}_* - V_1) = \rho_2(\dot{R}_* - V_2), \qquad \rho_2\left(1 - \frac{\rho_2}{\rho_1}\right)(\dot{R}_* - V_2)^2 = p_1 + \sigma_R. \quad (1)$$

In addition, it is assumed that at the wave front a phenomenological fracture criterion is satisfied, in the form of a certain linear relation between the principal stresses σ_R and $\sigma_\varphi = \sigma_\theta$:

$$F(\sigma_R, \sigma_\varphi) = m_1\sigma_R + m_2\sigma_\varphi - \sigma_0 = 0, \quad \text{for } R = R_*. \quad (2)$$

Here m_1, m_2, σ_0 are constants specified in accordance with the adopted theory of strength. For definiteness, we shall take the strength limit $\sigma_0 > 0$.

Let us specify the media under consideration:

1. Since we are dealing with brittle materials that fracture in the elastic stage of deformation, we shall assume that the motion of the strong medium is described by the wave equation

$$\frac{\partial^2 U}{\partial t^2} = a^2 \left(\frac{\partial^2 U}{\partial R^2} + \frac{2}{R}\frac{\partial U}{\partial R} - \frac{2U}{R^2} \right), \qquad a^2 = \frac{\lambda + 2\mu}{\rho_2}$$

(R is the initial coordinate of a particle–the Lagrangian variable), derived under the assumption that the particle displacements $U(t, R)$ are small and that the medium obeys Hooke' s law ([2]). The solution of this equation is

$$U(t, R) = -\frac{f'(\xi)}{R} - \frac{f(\xi)}{R^2}, \qquad \xi = at - R + R_{00}. \tag{3}$$

owing to the elastic wave expanding with the speed of sound a, depends on an arbitrary function $f(\xi)$, determined from the boundary conditions. Using (3), we obtain for the velocity, stresses, and fracture function the following expressions:

$$V_2 = \frac{\partial U}{\partial t} = -a \left(\frac{f''(\xi)}{R} + \frac{f'(\xi)}{R^2} \right),$$

$$\sigma_R = \lambda \left(\frac{\partial U}{\partial R} + 2\frac{U}{R} \right) + 2\mu \frac{\partial U}{\partial R} = (\lambda + 2\mu) \left[\frac{f''(\xi)}{R} + 4k^2 \left(\frac{f'(\xi)}{R^2} + \frac{f(\xi)}{R^3} \right) \right],$$

$$\sigma_\varphi = \lambda \left(\frac{\partial U}{\partial R} + 2\frac{U}{R} \right) + 2\mu \frac{U}{R} = (\lambda + 2\mu) \left[(1 - 2k^2) \frac{f''(\xi)}{R} - 2k^2 \left(\frac{f'(\xi)}{R^2} + \frac{f(\xi)}{R^3} \right) \right],$$

$$F = m_1 \sigma_R + m_2 \sigma_\varphi - \sigma_0 = (\lambda + 2\mu) \left[(m_1 + m_2(1 - 2k^2)) \frac{f''(\xi)}{R} + 2k^2(2m_1 - m_2) \left(\frac{f'(\xi)}{R^2} + \frac{f(\xi)}{R^3} \right) \right] - \sigma_0, \tag{4}$$

$$k^2 = \frac{\mu}{\lambda + 2\mu}.$$

2. To describe the motion of the destroyed, comminuted medium we shall use the equations of an ideal fluid ([3])

$$\frac{\partial \rho_1}{\partial t} + V_1 \frac{\partial \rho_1}{\partial \widetilde{R}} + \rho_1 \frac{\partial V_1}{\partial \widetilde{R}} + 2\frac{\rho_1 V_1}{\widetilde{R}} = 0,$$

$$\frac{\partial V_1}{\partial t} + V_1 \frac{\partial V_1}{\partial \widetilde{R}} + \frac{1}{\rho_1} \frac{\partial p_1}{\partial \widetilde{R}} = 0$$

($\widetilde{R}$ is the current coordinate of a particle—an Eulerian variable). Assuming that the comminuted medium is incompressible—which is natural, in view of the large pressures developed near the charge—we obtain the solution

$$\widetilde{R}^2 V_1 = g_1(t), \qquad \frac{p_1}{\rho_2} = -\frac{\dot{g}_1(t)}{\widetilde{R}} + \frac{g_1^2(t)}{2\widetilde{R}^4} = \frac{g_2(t)}{\rho_1},$$

depending on two arbitrary functions $g_1(t)$ and $g_2(t)$.

Fig. 2

3. Assuming that the charge detonates instantaneously, we shall consider the expansion of the material in the cavity as a gas with polytropic exponent γ:

$$p_0 = p_{00} \left(\frac{R_{00}}{\widetilde{R}_0} \right)^{3\gamma}.$$

Let us formulate the boundary conditions (Fig. 1):

a) at the boundary of the gas chamber with the comminuted medium $\widetilde{R}_0(t)$, the equalities hold

$$V_0 = V_1 = \dot{\widetilde{R}}_0, \qquad p_0 = p_1;$$

b) at the fracture wave, in addition to the aforementioned equations (1), (2), the following correspondence is established: $\widetilde{R}_* = R_*$ (owing to the smallness of the displacements in the elastic zone).

Fig. 3

c) at the front of the elastic wave the identities $\xi = 0$ and $U = 0$ hold, which, according to (3), gives the conditions

$$f(0) = f'(0) = 0. \tag{5}$$

These relations exhaust the formulation of the boundary-value problem.
Cauchy problem. In what follows, our aim will be to determine the function $R_*(t)$—the law of motion of the fracture wave. Reducing the number of equations and unknowns and passing from the independent variable t to ξ by means of the equality $\xi = at - R_*(t) + R_{00}$, we ultimately arrive at a system of three equations for determining the functions $f(\xi)$, $\widetilde{R}_0(\xi)$, $R_*(\xi)$:

$$\frac{f''}{R_*} = \frac{1}{m_1 + m_2(1 - 2k^2)} \left[\frac{\sigma_0}{\lambda + 2\mu} - 2k^2(2m_1 - m_2) \left(\frac{f'}{R_*^2} + \frac{f}{R_*^3} \right) \right],$$

$$\widetilde{R}_0' = \left(\frac{R_*}{\widetilde{R}_0}\right)^2 \left(1 - \frac{f'}{R_*^2} - \frac{f}{R_*^3}\right)^2 (a_2 R_*' + a_2)\,, \qquad a_i = a_i[f(\xi), f'(\xi), R_*(\xi)],$$

$$R_*'' = A_1 R_*'^3 + A_2 R_*'^2 + A_3 R_*' + A_4, \qquad A_i = A_i[f(\xi), f'(\xi), R_*(\xi), \widetilde{R}_0(\xi)],$$

where the coefficients $a_i(\xi)$, $A_i(\xi)$ are rather cumbersome and therefore are not written out here.

Let us formulate the initial conditions for this system. At $t = 0$ and $R = R_{00}$ we have $\xi = 0$, which gives 4 conditions:

$$R_*(0) = \widetilde{R}_0(0) = R_{00}, \qquad f(0) = f'(0) = 0.$$

The 5th condition is obtained from the equation of conservation of momentum:

$$R_*'(0) = \frac{\dot{R}_*(0)}{a - \dot{R}_*(0)}, \qquad \text{where} \quad \left(\frac{\dot{R}_*(0)}{a}\right) = \left[\frac{\frac{1}{1-\rho_2/\rho_1}\,\frac{\frac{p_{00}}{\lambda+2\mu} + \frac{\sigma_0}{\lambda+2\mu}\,\frac{1}{m_1+m_2(1-2k^2)}}{1+\frac{\sigma_0}{\lambda+2\mu}\,\frac{1}{m_1+m_2(1-2k^2)}}}{1}\right]^{1/2}.$$

Thus, the posed problem has been reduced to a Cauchy problem for a system of differential equations (whose total order is the 5th).

Existence of a fracture wave. Criterion (2) has a substantial influence on the motion. It is therefore meaningful to study the function F in the region ahead of the fracture wave (on the wave itself $F = 0$). In the plane σ_R, σ_φ, criterion (2) is represented by a straight line (Fig. 3). This straight line cannot pass through the origin $\sigma_R = \sigma_\varphi = 0$, since it lies in the region of strength. Substituting the coordinates of the origin into (4), we find that $F(0,0) < 0$ (since $\sigma_0 > 0$). Consequently, every stressed state satisfying the condition

$$F < 0, \tag{6}$$

will be sound, because the point representing it lies in the region containing the origin (Fig. 3). In the physical plane (Fig. 2), condition (6) must be satisfied everywhere in the region located between the R-axis and the curve $R_*(t)$. Fulfillment of the alternative condition $F > 0$ would mean the presence of fracture in this region, which contradicts the physical meaning of the model. Thus, the question of the existence of a fracture wave reduces to the problem of satisfying the inequality $F < 0$.

In the region of rest below the characteristic $\xi = 0$, where $\sigma_R = \sigma_\varphi = 0$, (6) is always satisfied. To prove this inequality in the zone occupied by elastic deformations (the hatched region in Fig. 2), let us trace the change of the function F along the characteristics $\xi = \text{const}$. On these characteristics the function $f(\xi)$ and its derivatives retain constant values, so that along some fixed ξ_i we have for F:

$$F\Big|_{\xi_i} = (\lambda + 2\mu)\left[(m_1 + m_2(1 - 2k^2))\frac{f''}{R} + 2k^2(2m_1 - m_2)\left(\frac{f'}{R^2} + \frac{f}{R^3}\right)\right] - \sigma_0. \tag{7}$$

At the point of intersection ξ_i with the curve $R_*(t)$ we obtain, according to (2) and (3),

$$f'' = \frac{R_*}{m_1 + m_2(1 - 2k^2)}\left[\frac{\sigma_0}{\lambda + 2\mu} - 2k^2(2m_1 - m_2)\left(\frac{f'}{R_*^2} + \frac{f}{R_*^3}\right)\right].$$

Substituting this quantity into (7), we find

$$F\Big|_{\xi_i} = -\sigma_0\left(1 - \frac{R_*}{R}\right) + \frac{2\mu(2m_1 - m_2)}{R}\left(-\frac{f'}{R_*} - \frac{f}{R_*^2} + \frac{f'}{R} + \frac{f}{R^2}\right) \tag{8}$$

or

$$F\Big|_{\xi_i} = -\sigma_0\left(1 - \frac{R_*}{R}\right) + 2\mu(2m_1 - m_2)\frac{U_* - U}{R}.$$

At the front of the elastic wave, which is usually the carrier of high stresses, $\xi = 0$, and, according to (5) and (8), we have

$$F\Big|_{\xi=0} = -\sigma_0\left(1 - \frac{R_{00}}{R}\right) < 0,$$

i.e., the strength condition (6) is necessarily satisfied. Since $F(t, R)$ is a continuous function, (6) is also satisfied in some small finite strip near the front of the elastic wave, in particular in a small region near R_{00} (Fig. 2). This thus proves the existence of a fracture wave at a moment close to the beginning of the motion ($t = 0,\ R = R_{00}$).

As for the fulfillment of (6) in the remaining part of the strength region, this depends on the second term on the right-hand side of (8). It is not difficult to see from (8) that, by an appropriate choice of m_1 and m_2 (which have not yet been determined), one can ensure satisfaction of (6). In particular, if we assume that $U_* - U$ is a positive quantity, then, choosing m_1 and m_2 so that $2m_1 - m_2 < 0$, we satisfy the strength condition. We note that the fracture function F used in this problem may be specified not only in the form (2), but also in the form of a more complicated function of σ_R and σ_φ. In that case only the first equation of the resulting system, which is the fracture equation, will change.

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Note: Figure translations are in progress. See original paper for figures.