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1963

SovietRxiv

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Abstract

Full Text

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RADIATION OF A PARTICLE FLYING PAST A WEDGE-SHAPED METALLIC SCREEN

(Presented by Academician M. A. Leontovich, January 9, 1963)

When a uniformly moving charge crosses the plane boundary separating two media (one of them, in particular, may be vacuum, the other a metal), transition radiation arises ([1]). If the boundary is not plane, radiation also arises when the charge moves past the boundary. The physical cause of the radiation is that the electromagnetic field of the moving charge produces a time-dependent polarization of the dielectric or induces time-dependent currents on the surface of the metal.

The most interesting case is that of a wedge-shaped boundary between a dielectric (vacuum) and a metal. As is known from diffraction theory, the component of the surface current parallel to the edge of the wedge diverges as the edge is approached; therefore the intensity of the radiation should increase, and the angular distribution should have a number of special features.

1. Let a medium with dielectric constant ε occupy the angular region $0 \leq \varphi \leq \gamma$ and be bounded by an ideally conducting wedge*. In the plane $z = 0$, perpendicular to the edge of the wedge, a particle moves at an angle α to the OX axis. Owing to the symmetry of the wedge, α may be restricted by the inequalities $0 \leq \alpha \leq \gamma - \pi$. The velocity of the particle is v, and at the moment $t = 0$ the particle is at the shortest distance a from the edge of the wedge (Fig. 1). On the faces of the wedge $\varphi = 0, \gamma$, the tangential components of the electric field must vanish.

Fig. 1. Plane $z = 0$; $OO' = a$; $O'P = x_0$

Let us introduce the electromagnetic potentials (Lorenz gauge) and expand all quantities in a Fourier integral with respect to time. When the particle moves perpendicular to the edge of the wedge, one may put $A_z \equiv 0$ and restrict oneself to transverse potentials. Then from the conditions for the fields $E_{\omega r} = E_{\omega z} = 0$ at $\varphi = 0, \gamma$, there follow the boundary conditions imposed on the potentials:

$$A_{\omega r} = 0; \qquad \frac{\partial A_{\omega\varphi}}{\partial \varphi} = 0 \quad \text{for } \varphi = 0, \gamma. \tag{1}$$

Fig. 2. Integration contours C_1, C_2 in (3)

Figure 2: Fig. 2. Integration contours C_1, C_2 in (3)

The field of a moving charge may be considered as a superposition of the fields of dipoles arranged along the trajectory of the charge ([3]). It is not difficult to show that if the potentials of a dipole $\mathbf{A}_\omega^d$ satisfying conditions (1) are known, then the components of the potentials of the moving particle satisfying

* The case of a thin plate ($\gamma = 2\pi$) was recently considered by A. P. Kazantsev and G. F. Surdutovich ([2]). Our results for $\gamma = 2\pi$ agree with the results of these authors.

under the same conditions, are determined by integration along the trajectory:

$$\mathbf{A}_\omega = -\frac{ie}{2\pi\omega\rho_0} \int_{-\infty}^{\infty} e^{\,i\frac{\omega}{v}x_0'}\, \mathbf{A}_\omega^{\mathrm{dip}}\, dx_0'. \tag{2}$$

In (2) a coordinate system has been used in which the motion of the charge takes place along the x' axis (Fig. 1).

The potentials of a dipole in a wedge-shaped region were found by G. D. Malyuzhinets and A. A. Tuzhilin ([4]) and by the author ([5]). Substituting them into (2) and carrying out the integration along the trajectory, we find the potentials of the moving charge

$$A_{\omega x,y} = \int_{C_1} du\, A(u)\, [\Gamma_{x,y}(u+\varphi-2\pi) \mp \Gamma_{x,y}(u-\varphi-2\pi)] -$$

$$- \int_{C_2} du\, A(u)\, [\Gamma_{x,y}(u+\varphi) \mp \Gamma_{x,y}(u-\varphi)]. \tag{3}$$

The integration contour is shown in Fig. 2. The notation is:

$$A(u) = \frac{ei}{4c\gamma} \exp\left[i\frac{\omega}{v} r\cos u\right] H_0^{(1)}\left(\frac{\omega}{v}T\right);$$

$$\Gamma_{x,y}(u) = \frac{\cos u}{\sin u}\left(1 - e^{\,\frac{i\pi}{\gamma}(u-\alpha)}\right), \tag{4}$$

$$T = \{(\varepsilon\beta^2 - 1)[(r\sin u + a)^2 + z^2]\}^{1/2}.$$

Fig. 2. Integration contours C_1, C_2 in (3)

$H_0^{(1)}$ is the Hankel function of the first kind. For $A_{\omega x}$ the minus sign is taken in the square bracket in (3), and for $A_{\omega y}$ the plus sign. $\operatorname{Im} T > 0$ on the branches of the integration contour going to infinity. In the range of angles $\alpha \leq \varphi \leq \pi+\alpha$, a term of the form

$$A^0_{\omega x,y} = \frac{ei}{2c}\frac{\cos\alpha}{\sin\alpha} e^{i\frac{\omega}{v} r\cos(\varphi-\alpha)} H_0^{(1)}\left(\frac{\omega}{v}\sqrt{(\varepsilon\beta^2-1)[(r\sin u + a)^2 + z^2]}\right),$$

which gives the Fourier component of the potentials of the particle in free space, must be added to the expression for the potentials (3).

2. We shall restrict ourselves to investigating the case $\varepsilon = 1$, when the radiation is completely determined by currents induced on the surfaces of the plates. The field components in the wave zone are not difficult to calculate by the saddle-point method, replacing the Hankel function in (3) by its asymptotic representation. The energy radiated by the charge during its passage is equal to

$$E = \int d\Omega \int_{-\infty}^{\infty} d\omega\, W_\omega(\theta,\varphi). \tag{5}$$

The integration over $d\Omega$ is carried out over the part of the surface of a sphere of large radius bounded by the planes $\varphi = 0, \gamma$. Polar coordinates are used, with the polar axis directed along the edge of the wedge (the z axis),

$$W_\omega(\theta,\varphi) = \frac{e^2 \exp\left[-\dfrac{2a|\omega|\sqrt{1-\beta^2\sin^2\theta}}{v}\right]}{8\gamma^2 c\sin^2\theta}\left\{\frac{\cos^2\theta}{1-\beta^2\sin^2\theta}[\Phi_1(i\tau-2\pi)-\Phi_1(i\tau)]\times\right.$$

$$\times[\Phi_1^*(i\tau-2\pi)-\Phi_1^*(i\tau)] + [\Phi_2(i\tau-2\pi)-\Phi_2(i\tau)]\times$$

$$\left.\times[\Phi_2^*(i\tau-2\pi)-\Phi_2^*(i\tau)]\right\}, \tag{6}$$

where

$$e^\tau = \left(1+\sqrt{1-\beta^2\sin^2\theta}\right)/\beta\sin\theta;$$

$$\Phi_{1,2}(u) = \frac{1}{1-e^{-i\frac{\pi}{\gamma}(u+\varphi-\alpha)}} \mp \frac{1}{1-e^{-i\frac{\pi}{\gamma}(u-\varphi-\alpha)}}.$$

Formulas (5)–(7) make it possible to indicate the following properties of the type of radiation under consideration.

The radiation is predominantly long-wavelength. Wavelengths

$$\lambda \gtrsim 2\pi a\sqrt{1-\beta^2\sin^2\theta}/\beta$$

are represented with appreciable intensity. In the ultrarelativistic case ($1-\beta \ll 1$) the characteristic wavelength may become considerably smaller than the distance to the edge of the wedge a.

The intensity of radiation of frequency ω per unit solid angle $W_\omega(\theta,\varphi)$ has a characteristic singularity: $W_\omega(\theta,\varphi)$ diverges as $\theta \to 0, \pi$ proportionally to $(\sin\theta)^{2\pi/\gamma-2}$. This singularity is connected with the fact that the transverse components of the field are large in the vicinity of the edge, so that for $\beta \ll 1$ the radiation is concentrated in a narrow cone about the edge of the wedge.

In the nonrelativistic case, in the denominators of the functions $\Phi_{1,2}$ one may neglect unity in comparison with the exponential. Then

$$W_\omega(\theta,\varphi) \simeq 2e^2\left(\frac{\beta}{2}\right)^{2\pi/\gamma}(\sin\theta)^{2\pi/\gamma-2}\exp\left(-\frac{2a|\omega|}{v}\right)\times$$

$$\times\left[\sin^2\left(\frac{\pi}{\gamma}\varphi\right)\cos^2\theta+\cos^2\left(\frac{\pi}{\gamma}\varphi\right)\right]$$

and the total radiated energy is

$$E \simeq A(\gamma)\frac{e^2}{a}\left(\frac{\beta}{2}\right)^{2\pi/\gamma+1}\sin^2\left(\frac{\pi^2}{\gamma}\right). \tag{8}$$

Here $A(\gamma)$ is a factor of order unity. As the angle γ changes from 2π (thin plate) to π, the exponent of β changes from 2 to 3, i.e., the influence of the edge disappears and the radiation becomes "dipole" radiation, decreasing in absolute value as $\sim\sin^2(\pi^2/\gamma)$.

In the ultrarelativistic case the angular distribution has two maxima. The maxima correspond to those angles θ,φ for which the denominators in the functions $\Phi_{1,2}$ are small, i.e., the vicinities of $\theta=\pi/2$; $\varphi=\alpha$ and $\varphi=2\gamma-2\pi-\alpha$. The first of them, as was to be expected, coincides with the direction of motion of the particle; the second is determined by the direction of photons reflected according to the laws of geometrical optics from the nearest plate $\alpha=\gamma$. For $\alpha=\gamma-\pi$ the two directions coincide.

Let us finally turn to the total radiation energy E. Integration over ω and φ in (5) gives

$$E = \frac{e^2\beta}{\gamma\omega}\int_0^{\pi/2}\frac{d\theta}{\sin\theta\sqrt{1-\beta^2\sin^2\theta}}\left[1+\frac{\cos^2\theta}{1-\beta^2\sin^2\theta}\right]\times$$

$$\times\left\{\frac{1}{e^{\frac{2\pi}{\gamma}\tau}-1}-\frac{e^{\frac{2\pi}{\gamma}\tau}\cos\left(\frac{2\pi^2}{\gamma}\right)-1}{1-2e^{\frac{2\pi}{\gamma}\tau}\cos\left(\frac{2\pi^2}{\gamma}\right)+e^{\frac{4\pi}{\gamma}\tau}}\right\}. \quad (9)$$

Thus, for any particle velocity the total radiation does not depend on the angle of flight, as in the case of a plate [(2)]. In the ultrarelativistic case the main contribution to the radiation is made by the vicinity of $\theta = \pi/2$ and

$E \simeq B(\gamma)\dfrac{e^2}{a\sqrt{1-\beta}}$, $B(\gamma) \sim 1$. The total radiation energy is proportional to the energy of the particle.

The author expresses his gratitude to A. M. Dykhne, V. L. Pokrovskii, Yu. B. Rumer, A. P. Kazantsev, and G. F. Surdutovich for discussion of the work.

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Received
8 I 1963

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Note: Figure translations are in progress. See original paper for figures.