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Abstract

Full Text

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THEORY OF ELASTICITY

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POSTCRITICAL DEFORMATIONS OF BOUNDEDLY ELASTIC SHELLS

1. Loss of stability and transition to postcritical deformations of a geometrically nonbendable shell is accompanied by the appearance on its surface of sharply distinguished edges, which are smoothed singularities of the isothermally transformed middle surface. The point is that a shell, being geometrically nonbendable, does not admit isometric transformations without singularities. At the same time, under ordinary deformations of the shell material, considerable changes in its external form can be conceived only as deformations close to isometric ones. And since isometric transformations for our shell are possible only with singularities, the transition of the shell to the postcritical state, associated with a considerable change in external form, is accompanied by the appearance on its surface of the above-mentioned edges (see ([1])).

2. The appearance of edges on the surface of the shell under postcritical deformation causes considerable deformations of the shell material—bending in the plane perpendicular to the edge, and the stretching and compression in the middle surface in the direction of the edge that accompany this bending. The stresses arising in this case prove to be so considerable that real shells, as a rule, after loss of stability undergo elastic-plastic deformations. This circumstance cannot be ignored. In questions connected with postcritical deformations, real shells, possessing bounded elasticity, must be identified with ideal, i.e. unboundedly elastic, shells only with great caution.

3. The formation of edges on the surface of a shell occurs at the moment of loss of stability. The postcritical deformation proper is accompanied only by a change in the form of the edges. If attention is fixed on an element of an edge, then its deformation is perceived as a displacement. The passage of the edge through a given point of the shell surface in the course of its displacement is accompanied by considerable bending of the shell, first in one direction and then in the opposite one, practically restoring the initial form. If the normal curvature in the section perpendicular to the edge is large, then the energy of such a deformation, associated with the displacement of the edge, depends essentially on the elastic limit of the material and will be the greater, the lower the elastic limit. Thus, the displacement of edges, and consequently also the postcritical deformation of boundedly elastic shells, is impeded in comparison with ideal, i.e. unboundedly elastic, shells.

4. In the author' s work ([1]) the form of the shell near an edge was found as a function of its principal geometric parameters and of the modulus of elasticity of the material. The shell was assumed to be unboundedly elastic. We have now considered this problem for shells possessing bounded elasticity, under the condition that the state diagram of the material in the region of plastic deformations is a straight line parallel to the axis of deformations. The result obtained in this case may be formulated as the following

in this way. If, as a result of postcritical deformation of the shell, an edge arises on its surface and the bending caused by its appearance leads to plastic deformations, then the form of the shell surface near the edge becomes unstable and changes into one for which the radius of curvature in the direction perpendicular to the edge becomes close to zero. Hence we draw the important conclusion that **the edge on the surface of the shell at the moment when bending stresses exceeding the elastic limit appear is fixed**, since displacement of such an edge is associated with a very large (theoretically infinitely large) deformation energy.

5. The form of the surface under postcritical deformation of a geometrically non-bendable shell is, in essence, determined by its edges. Therefore the determination of the form of the shell in a state of equilibrium, based on energy considerations, is connected with varying only the form of the edges. The conclusion obtained in Sec. 4 allows us to conclude that, for shells possessing limited elasticity, this variation of the edges must be subordinated to an additional condition, namely: **the bending stresses along the edges must not exceed the σ-elastic limit of the material.**

6. In the work ([1]) the question of the magnitude of the lower critical pressure q_k on a strictly convex shell fixed along its boundary was considered. It was shown there that determining the magnitude of this pressure is impossible without taking into account the limited elasticity of the material, and for the magnitude q_k the estimate was obtained

$$q_k \leqslant 3cc' E\,(K\delta^2)(H\delta)\,\frac{E}{\sigma}.$$

Here H and K are, respectively, the mean and Gaussian curvatures of the shell surface; δ is the thickness; E is the modulus of elasticity; σ is the temporary resistance (also the elastic limit); c and c' are constants. In the work the question was discussed whether the value on the right-hand side of the inequality could be considered close to the magnitude q_k. The solution of this question depended on the magnitude of the energy of elastic-plastic deformation of the shell during motion of the edge. Now, when it has been clarified that this energy is very large, theoretically infinitely large, **one may consider**

$$q_k = 3cc'E\,(K\delta^2)(H\delta)\,\frac{E}{\sigma},$$

in particular, for a spherical shell

$$q_k = 3cc'E\left(\frac{\delta}{R}\right)^3\frac{E}{\sigma}.$$

Let us note that **the application of this formula, like that of the preceding one, is limited by the condition that the shell must be sufficiently thin, namely**

$$\delta/R < \sigma_e/E.$$

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REFERENCES

1. A. V. Pogorelov, *On the Theory of Convex Elastic Shells in the Postcritical Stage*, Publishing House of Kharkov State University, 1960.

Note: Figure translations are in progress. See original paper for figures.