Ya. I. VIZBARAITE, Z. B. RUDZIKAS, Academician of the Academy of Sciences of the Lithuanian SSR A. P. YUTSIS

1963

SovietRxiv

Abstract

Full Text

ASTRONOMY

Ya. I. VIZBARAITE, Z. B. RUDZIKAS, Academician of the Academy of Sciences of the Lithuanian SSR A. P. YUTSIS

THEORY OF NEBULAR FORBIDDEN LINES CORRESPONDING TO MAGNETIC MULTIPOLE TRANSITIONS

In the spectra of planetary nebulae and of the solar corona there are forbidden lines. All these lines are often called nebular lines. The term forbidden lines means that they cannot be attributed to electric dipole transitions. Some forbidden lines have been attributed to electric quadrupole and magnetic dipole transitions. However, a considerable part of the forbidden lines is not covered by theory.

Intercombination lines, corresponding to transitions between terms of different multiplicities of different configurations, can be covered by known radiation operators when the deviation from LS-coupling is taken into account ([1]); however, many of the nebular lines correspond to transitions between terms of different multiplicities of one and the same configuration, in which LS-coupling gives a good characterization of the states. It is precisely these that constitute one category of lines not covered by theory.

On the other hand, among the nebular forbidden lines there are also such lines as correspond to transitions between levels for which the difference of the quantum numbers exceeds the values allowed by the corresponding theory. These lines are also not covered by the existing theory.

The facts cited indicate the necessity of finding ways to extend the existing theory so that it would cover the forbidden lines mentioned and would make it possible to predict as yet unknown lines, which is especially important from the practical point of view. The present note is devoted to this question.

We shall restrict ourselves to considering the question of the possibility of constructing a magnetic multipole operator of such rank and such structure that the corresponding selection rules could cover all forbidden nebular lines. We shall solve this problem with the aid of a generalized orbital angular-momentum operator, combining it with the spin angular momentum.

Let us have an irreducible tensor operator of rank k, having the structure

$$l^{(k)} = [l^{(1)} \times l^{(1)} \cdots \times l^{(1)}]^{(k)}, \tag{1}$$

where $l^{(1)}$ is the operator of the orbital angular momentum, specified by its components in spherical coordinates. The square brackets denote the tensor product of k factors equal to one another. It should be noted that the number of factors must not exceed the rank of the tensor product, since otherwise the permutation relations of the angular momentum may be applied, reducing the number of factors to the rank of the product.

The operator (1) is a generalization of the angular-momentum operator. Apart from a factor depending on the one-electron quantum numbers l, it coincides with Racah' s operator $u^{(k)}$ ([2,3]). The rank k cannot exceed $2l$, since for $k > 2l$ its matrix element is equal to zero. We note that what is meant here is a shell consisting of equivalent electrons.

The spin angular-momentum operator $s^{(1)}$ is not subject to generalization, since its rank is limiting for the spin quantum number $s = 1/2$ $(2s = 1)$.

Using the operators $l^{(k)}$ and $s^{(k)}$, we construct a new operator

$$w^{(k1k')} = [\, l^{(k)} \times s^{(1)} \,]^{(k')}, \tag{2}$$

where k' may take the values $k-1, k, k+1$, since $l^{(k)}$ and $s^{(1)}$ commute with one another. The operator $w^{(k1k')}$, with respect to submatrix elements, coincides with the operator $V^{(1k)}$, introduced in ([2,3]), up to the same multiplier by which $l^{(k)}$ coincides with $u^{(k)}$.

It is natural to assume that the operator of the magnetic $2^{k'}$-pole for a shell consisting of equivalent electrons has the form

$$M^{(k1k')} = a \sum_j w_j^{(k1k')}, \tag{3}$$

where a is a constant independent of the characteristics of the states of the atomic electrons and containing the charge and mass of the electron, as well as the speed of light. For determining relative intensities, an explicit expression for this constant is unnecessary.

If one takes the matrix element of the operator (3), applies the Eckart–Wigner theorem, squares it, and performs summation over magnetic quantum numbers and projections of rank k', one obtains

$$S\,(l^N \alpha LSJ,\; l^N \alpha' L' S' J') = \left| \left(l^N \alpha LSJ \middle\| M^{(k1k')} \middle\| l^N \alpha' L' S' J' \right) \right|^2 . \tag{4}$$

On the left-hand side of this equality stands the total line strength, while for the submatrix element on the right-hand side, according to formula (35.3) ([4]), we have

$$\left(l^N\alpha LSJ\left\|M^{(k1k')}\right\|l^N\alpha' L'S'J'\right)$$
$$= [(2J+1)(2J'+1)(2k'+1)]^{1/2}\left\{\begin{array}{ccc} L & S & J \\ L' & S' & J' \\ k & 1 & k' \end{array}\right\}\left(l^N\alpha LS\left\|M^{(k1)}\right\|l^N\alpha' L'S'\right). \qquad (5)$$

The submatrix element on the right-hand side of the last equality can be expressed by the formula

$$\left(l^N\alpha LS\left\|M^{(k1)}\right\|l^N\alpha' L'S'\right) =$$

$$= a[\,l(l+1)(2l+1)\,]^{k/2}\left(l^N\alpha LS\left\|V^{(k1)}\right\|l^N\alpha' L'S'\right). \qquad (6)$$

The values of the submatrix element on the right-hand side of equality (6) for p- and d-electrons are completely determined and are given in ([3,5,6]).

The conditions for nonvanishing of the $9j$-coefficient on the right-hand side of (5) give the selection rules for a transition of the magnetic $2^{k'}$-pole within the configuration l^N. They require that the triangles

$$\{LL'k\}, \qquad \{SS'1\}, \qquad \{JJ'k'\}. \qquad (7)$$

be satisfied. Extensive material concerning existing forbidden transitions within p- and d-shells is presented in ([7]). All these transitions fit the selection rules (7), if

$$k = 1, \ldots, 2l, \qquad k' = k,\ k-1. \qquad (8)$$

Application of the above-mentioned tables of submatrix elements of the operator $V^{(k1)}$ makes it possible to determine the relative intensities of all these lines from formulas (4)–(6).

Institute of Physics and Mathematics
Academy of Sciences of the Lithuanian SSR

Vilnius State University
named after V. Kapsukas

Received
6 X 1962

References

1. Ya. I. Vizbaraite, V. I. Chiplis, A. P. Yutsis, *DAN*, **135**, 1101 (1960); *Tr. AN LitSSR*, B, **2**(25), 11 (1961).

2. G. Racah, *Phys. Rev.*, **62**, 438 (1948).

3. G. Racah, *Phys. Rev.*, **63**, 367 (1943).

4. A. P. Yutsis, I. B. Levinson, V. V. Vanagas, *Mathematical Apparatus of the Theory of Angular Momentum*, Vilnius, 1960.

5. J. C. Slater, *Quantum Theory of Atomic Structure*, **2**, N. Y., 1960.

6. Z. B. Rudzikas, Ya. I. Vizbaraite, A. A. Bandzaitis, A. P. Yutsis, *Tr. AN LitSSR*, B, **4**(31), 3 (1962).

7. I. S. Bowen, *Appl. J.*, **132**, 1 (1960).

Note: Figure translations are in progress. See original paper for figures.