SOLUTION OF THE PROBLEM OF THE MOTION OF INTERACTING CHARGED PARTICLES OF ONE SIGN IN A "MAGNETIC BOTTLE" ON ELECTRONIC COMPUTERS

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SOLUTION OF THE PROBLEM OF THE MOTION OF INTERACTING CHARGED PARTICLES OF ONE SIGN IN A “MAGNETIC BOTTLE” ON ELECTRONIC COMPUTERS

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In some cases the problem of the motion of a large number of interacting charged particles can be approximated by a system of N equations of motion with interaction terms for N disks (1), N “enlarged” particles (2,3), or N points of observation (4). Such a formulation has a number of advantages over the scheme of the Boltzmann kinetic equation, since it makes it possible to formulate the initial-value problem in a natural way and to take into account the practical conditions of injection of particles into the region occupied by the external electric and magnetic field.

It should be noted that the solution of these equations on high-speed computers resembles an experiment on the corresponding real installation, both in the unfolding of the process in time and in the possibility of varying the external fields, the injection conditions, and other characteristics of the installation.

In (1) a first step was made in the study of plasma: a one-dimensional problem of the motion of charged disks of different signs was solved without taking account of an external field. As follows from that work, such an approach is not entirely justified, since it does not make it possible to establish the very important relationship between the effect of Coulomb repulsion and the confinement of particles by an external field, as well as the connection between the longitudinal and transverse motion of the particles.

Below a calculation is proposed for the motion of enlarged charges, with allowance for interaction, in a system of the “magnetic bottle” type. For simplicity, charged particles of one sign are taken, although in principle systems with charges of both signs can be considered.

The following methods were used for solving the problem and specifying the initial conditions:

I. The natural method (determined by the conditions of continuous injection of particles into the magnetic field), in which a time-sequential set of the j-th enlarged charges is taken (analogously, for points of observation (4)) with step

Fig. 1. Example of a section of rings by the rz plane when the initial values are specified by method II. $\tau = 0.4$; a—$q = 0.5$; —$q = 0$.

Figure 1: Fig. 1. Example of a section of rings by the rz plane when the initial values are specified by method II. $\tau = 0.4$; a—$q = 0.5$; —$q = 0$.

Δt, and having, at $t = j\Delta t$ ($j = 0, 1, \dots, N$), identical coordinates and velocities. The choice of Δt and N is determined by the capabilities of the computer, the time, and the injection power. Interrupting the injection at the time $T = \Delta t\, N$ and fixing the positions and velocities of the j-th particles, we obtain the initial conditions for the system of equations describing the motion of N interacting material points. The equations of motion in the cylindrical coordinate system r, φ, z will be:

$$\ddot{r}_j = v_{\varphi j}^2 / r_j + v_{\varphi j} H_{zj} + \varepsilon_{rj},$$

$$\ddot{z}_j = -v_{\varphi j} H_{rj} + \varepsilon_{zj}, \tag{1}$$

$$\ddot{\varphi}_j = \frac{1}{r_j}\left(\dot{z}_j H_{rj} - \dot{r}_j H_{zj} - 2\dot{\varphi}_j \dot{r}_j + \varepsilon_{\varphi j}\right),$$

$$v_{\varphi j} = r_j \dot{\varphi}_j, \qquad j = 0, 1, \dots, N,$$

where differentiation has been carried out with respect to $\tau = t/t_0$; H_r and H_z are the components of the magnetic-field strength, which may, for example, be taken in the form

$$H_r = \beta I_1(r) \sin z, \qquad H_z = \alpha + \beta I_0(r) \cos z; \tag{2}$$

α and β are constants of the installation. $\varepsilon_{rj}, \varepsilon_{zj}$, and $\varepsilon_{\varphi j}$ are found as the projections of the expression

$$\bar{\varepsilon}_j = q_j \sum_{\substack{i \\ i \neq j}}^{N} \frac{q_i \bar{R}_{ij}}{R_{ij}^3}; \tag{3}$$

(q_j is the j-th charge) onto the corresponding axes. When using the mean-density method, $\bar{\varepsilon}_j$ is constructed according to (4).

Fig. 1. Example of a section of rings by the rz plane when specifying the initial values by method II. $\tau = 0.4$; a—$q = 0.5$;
—$q = 0$.

Figure 2

Figure 2: Figure 2

II. Other methods were based on the assumption of a certain initial distribution of particles independent of φ. The charge was uniformly smeared over φ. Taking into account that the magnetic-field components H_r and H_z do not depend on φ, we obtain a solution independent of φ. Therefore we shall consider a two-dimensional problem for the coordinates r and z, carrying out integration over φ, i.e., writing the equations of motion for geometrical manifolds with coordinates $r_j, z_j, 0 \leqslant \varphi_j \leqslant 2\pi$, which are rings, where j is the index enumerating the rings. Each ring (this is one enlarged particle) is described by the first two equations of system (1), and

$$v_{\varphi j} = \frac{D_j}{r_j} - \left(\frac{\alpha}{2} r_j + \beta I_1(r) \cos z_j\right). \tag{4}$$

D_j is found from the initial conditions

$$\varepsilon_{rj} = \frac{q_j}{2\pi} \sum_{\substack{i \\ i \neq j}}^{N} \int \frac{n_i (r_j - r_i)}{R_{ij}^3} \, d\varphi,$$

$$\varepsilon_{zj} = \frac{q_j}{2\pi} \sum_{\substack{i \\ i \neq j}}^{N} \int \frac{n_i (z_j - z_i)}{R_{ij}^3} \, d\varphi; \tag{5}$$

n_i is the distribution of the density of the i-th ring with respect to φ, equal to $q_i/2\pi$. The terms with $\dot{r}, \dot{z}, \ddot{r}, \ddot{z}$ in (3) and (5) are omitted, since they have no appreciable influence on the solution of (1).

The assumption of symmetry with respect to φ greatly reduces the number of equations that must be considered; at the same time, because of the limited memory of the machine, the possibilities for analyzing the entire device are appreciably increased.

It is convenient to choose the initial values by one of the following methods:

a) Taking $r_{0j}, z_{0j}, \dot{r}_{0j}, \dot{z}_{0j}$ from a set of points arranged successively in t with step Δt and obtained from the solution of (1) for $N = 0$ and initial values corresponding to the injection conditions.

b) Using the solution of (1) for $N = 0$ and initial conditions corresponding to the injection conditions, dividing the region in which the solution r, z exists by a uniform grid, averaging $r_{0j}, z_{0j}, \dot{r}_{0j}, \dot{z}_{0j}$ over the grid cells, and assigning the resulting values to the center of the cell.

Fig. 3 and Fig. 4

Figure 3: Fig. 3 and Fig. 4

Fig. 2. Set rz under the natural assignment of initial values
$\tau = 1.1$; $a - q = 0.5$; $b - q = 0$

In both cases, after assigning the initial values, the process is referred to a single instant of time $\tau = 0$; $r_{0j}, z_{0j}, \dot{r}_{0j}, \dot{z}_{0j}$ are the initial values for any point of the j-th ring.

The problem was solved on the BESM of the Academy of Sciences of the USSR with parameters of practical interest: $\alpha = 10$; $\beta = 2$; $\tau = 10^{-8}$, and with various methods of assigning the initial values.

Of course, because of the limited capabilities of the machine ($N_{\max} < 500$), complete identity of the solutions for the different methods of assigning the initial values could not be achieved, but the general result and the regularities are the same:

1. The longitudinal motion of the particles in the center of the device leads to expansion of the beam in the transverse direction and to the loss of part of it on the walls (Fig. 1).
2. Compression of the particles by the field in the region of the "magnetic mirrors" ($z = 0; 2\pi$) leads to their penetration through the magnetic mirrors and to losses (Fig. 2).

One or another character of the particle losses depends on the values of α, β and on the initial values.

At the same time, a statistical processing of the results was carried out for different values of q (Fig. 3), which made it possible to conclude:

1. $\bar{r}(\tau)$, the mean value of r, and $\delta r(\tau)$ and $\delta z(\tau)$, the dispersions in r and z, grow proportionally to $\sqrt{\tau}$ (Fig. 3).
2. $\bar{z}(\tau)$—the mean value of z—is equal to a constant.
3. The energy distribution is not Maxwellian (Fig. 4).

The smoothness of $\bar{r}(\tau)$, $\delta r(\tau)$, and $\delta z(\tau)$ up to the τ values indicated in Fig. 3 could be established only for $N = 100$. Thereafter the dependences had an oscillatory form, and with increasing N smoothing occurred. The smoothness of $\bar{r}(\tau)$, $\delta r(\tau)$, and $\delta z(\tau)$ can be used as an indirect indication of the adequacy of the chosen N. It is true that some oscillations of these dependences

Fig. 3. Example of the dependences $\bar{r}(\tau)$, $\delta r(\tau)$, $\delta z(\tau)$.
1—$q = 0.5$; 2—$q = 0.25$; 3—$q = 0.1$; 4—$q = 0$

Fig. 4. Example of the distribution of particles by velocities for $q = 0.1$.
1—$\tau = 0.5$; 2—$\tau = 1.0$; 3—$\tau = 1.5$; 4—$\tau = 2.0$

do not disappear upon an additional increase of N (the dashed part of curve 2 in Fig. 3). Evidently, in this case we are dealing with a local phenomenon of elastic expansion of the region filled with particles, owing to their drift in both directions along z, analogous to the excitation of radial instabilities by phase oscillations in accelerators ([3]).

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Note: Figure translations are in progress. See original paper for figures.