FREQUENCY SHIFT IN THE SCATTERING OF LIGHT IN A RELATIVISTIC PLASMA

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Abstract

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O. POGUTSE

FREQUENCY SHIFT IN THE SCATTERING OF LIGHT IN A RELATIVISTIC PLASMA

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The paper considers the scattering of light by an isotropic plasma with a Maxwell distribution function. It is shown that, when relativity is taken into account, the maximum of the scattered light is displaced into the region of higher frequencies. In the case of strong relativity, the relative frequency shift corresponding to the maximum of the scattered radiation is of order $1/(1-\beta_T)$. The results obtained may be of interest for diagnostics of high-temperature plasma.

In the scattering of light (i.e., electromagnetic waves of sufficiently high frequencies), collective effects in the plasma may be neglected (i.e., the scattering charges may be regarded as noninteracting). For this it is sufficient that the condition $\lambda \ll d$ be satisfied (λ is the wavelength of the incident radiation, d is the Debye radius). If this condition is rewritten in terms of frequencies, it takes the form $\Omega \gg \omega_0/\beta$ (Ω is the frequency of the incident radiation, ω_0 is the electron plasma frequency, $\beta = v_T/c$; $v_T = \sqrt{2T/mc^2}$). For $\beta \sim 1$ this condition reduces to $\Omega \gg \omega_0$, while collective effects in a relativistic plasma can affect the scattering only in the vicinity of the plasma frequency.

General formulas for the scattering of electromagnetic waves in a plasma are given in ([1]). In the case of noninteracting particles they take the form:

$$\sigma = \int do\, d\omega\, \tilde{\sigma} = \int do\, d\omega\, k^2\, dk\, \frac{1}{n}\, \frac{(2\pi)^3}{\omega}\, \delta\left[\left(\frac{kc}{\omega}\right)^2 - 1\right] \times$$

$$\times \left(\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}\right) (\Delta I_\alpha \Delta I_\beta^*)_{k\omega}\, \frac{c}{8\pi}\, (E_1^0, E_1^{0*}),$$

$$(\Delta I_\alpha \Delta I_\beta^*)_{k\omega} = (B_{\alpha\gamma} B_{\beta\delta}^*)_{k\omega} E_{1\gamma}^0 E_{1\delta}^{0*}, \tag{1}$$

$$(B_{\alpha\gamma} B_{\beta\delta}^*)_{k\omega} = \left(\frac{e^2}{2}\right)^2 \int \frac{dp}{(2\pi)^3}\, [\delta(\omega - \Omega - (k-\chi)v) +$$

$$+ \delta(\omega + \Omega - (k+\chi)v)]\, f_0(p)\, [\sigma\gamma][\lambda\delta]\, \frac{c^4}{\varepsilon^2} \times$$

$$\times \frac{[\delta_{\rho\sigma} - \beta_\rho \beta_\sigma][\delta_{\pi\lambda} - \beta_\pi \beta_\lambda]}{(\omega - kv)^2} \left[\delta_{\alpha\rho} + \frac{v_\alpha k_\rho}{\omega - kv}\right] \left[\delta_{\beta\omega} + \frac{v_\beta k_\pi}{\omega - kv}\right],$$

$$[\alpha\beta] = \left[\left(1 - \frac{\chi v}{\Omega}\right) \delta_{\alpha\beta} + \frac{\chi_\alpha v_\beta}{\Omega}\right].$$

Let us first consider the case of a weakly relativistic Maxwellian plasma ($\beta \ll 1$).

Keeping in (1) only the terms linear in β and averaging the expression obtained over the Maxwell distribution, we obtain the following result for the differential scattering cross section:

$$\tilde{\sigma} = \tilde{\sigma}_0 \left(1 + 2\frac{\omega - \Omega}{\Omega}\right) (1 - \sin^2\theta \cos^2\varphi); \tag{2}$$

θ is the angle between k and $\varkappa$, φ is the angle between the projection of k onto the plane perpendicular to $\varkappa$ and E^0; E^0 is the amplitude of the incident wave; ω is the frequency of the scattered light, Ω that of the incident light; $\tilde{\sigma}_0$ is the scattering cross section for $\beta = 0$, and has the form:

$$\tilde{\sigma}_0 = \left(\frac{e^2}{mc^2}\right)^2 \frac{e^{-\tilde{\omega}^{2/k^2} v_T^2}}{\pi^{1/2} v_T k}, \qquad \tilde{\omega} = \omega - \Omega, \qquad \tilde{k} = \sqrt{k^2 + \varkappa^2 - 2k\varkappa\cos\theta}. \tag{3}$$

A formula of the form (2) was obtained in the work ([2]), but there the second term contains the coefficient 3 instead of 2. It can be shown that this occurs because the authors of ([2]) did not take into account the difference between the time in the Liénard-Wiechert potentials and the time t' in the laboratory system. Namely, these times are related by

$$dt = \left(1 - \frac{nv}{c}\right) dt'$$

(see formula (72,8) in ([3])). Owing to this difference, one more term of order β appears in the cross section. Taking it into account leads to formula (2).

As follows from formula (2), the angular dependence of the cross section does not change when terms of order β are included, while the maximum of the scattered radiation is shifted toward higher frequencies by the amount

$$\Delta\omega = \omega - \Omega = 2\Omega\beta_T^2(1 - \cos\theta), \qquad \beta_T^2 = \frac{2T}{mc^2}. \tag{4}$$

Let us now consider the case of strong relativism ($T \gg mc^2$). To simplify the calculation, we shall restrict ourselves to the particular case of scattering through the angle $\theta = \pi/2$ (θ is the angle between the direction of the incident beam and the direction of observation). Let E^0 be directed along the x-axis, k, the wave vector of the scattered wave, along the y-axis, and $\varkappa$, the wave vector of the incident wave, along the z-axis. In this case formula (1) takes the form:

$$\sigma = \int do\, d\omega\, \tilde{\sigma},$$

$$\tilde{\sigma} = \frac{1}{2}\left(\frac{e^2}{mc^2}\right)^2 m^2 \int dp\, \{\delta[\omega(1-\beta_y) - \Omega(1-\beta_z)] + \delta[\omega(1-\beta_y) + \Omega(1-\beta_z)]\}$$
$$\times \frac{c^4}{\varepsilon^2}\frac{1}{(1-\beta_y)^2}\left[\left(1-\beta_z-\frac{\beta_x^2}{1-\beta_y}\right)^2 + \left((1-\beta_y-\beta_z)^2\frac{\beta_x^2}{(1-\beta_y)^2}\right)\right] f_0(p),$$
$$f_0(p) = Ae^{-\varepsilon/T}, \qquad \varepsilon = mc^2\sqrt{1+\left(\frac{p}{mc}\right)^2}; \tag{5}$$

A is a normalization constant:

$$A = \frac{1}{4\pi(mc)^3}\frac{1}{2\left(\frac{T}{mc^2}\right)^2 K\left(\frac{mc^2}{T}\right) + \frac{T}{mc^2}K_0\left(\frac{mc^2}{T}\right)}; \tag{6}$$

$$\beta_y = \beta\sin\theta\sin\varphi, \qquad \beta_z = \beta\cos\theta, \qquad \beta_x = \beta\sin\theta\cos\varphi,$$

θ, φ are angles in momentum space. It is easy to integrate over $d\varphi$ by means of the δ-function. The remaining integrals over $d\theta$ and dp are very complicated. To simplify the integration, let us consider the most interesting case $\omega \gg \Omega$ (as will be seen below, the maximum of the scattering cross section lies in this frequency region). Using the condition $\omega \gg \Omega$, one can carry out the integration over $d\theta$ in (5), and from the condition that the limits of integration be real one obtains a restriction on the frequencies. Namely, the integral over $d\theta$ is equal to zero in the case

$$\omega > \frac{\Omega}{1-\beta}$$

and is equal to

$$\tilde{\sigma} = \left(\frac{e^2}{mc^2}\right)^2 \int p^2\, dp \left(\frac{mc^2}{\varepsilon}\right) f_0(p)\,\frac{4}{\omega}\left(\frac{\omega}{\Omega}\right)^2 \frac{\pi}{2}\left(1 - 2\left[\frac{\omega}{\Omega}(1-\beta) - \left(\frac{\omega}{\Omega}\right)^2(1-\beta)^2\right]\right) \tag{7}$$

in the case

$$\omega \leqslant \frac{\Omega}{1-\beta}.$$

If β is expressed in terms of the energy ε, this condition can be rewritten in the form

$$\varepsilon \gg mc^2 \sqrt{\frac{\omega}{2\Omega}},$$

i.e., $mc^2\sqrt{\omega/2\Omega}$ is the lower limit in the energy integral.

As a result we obtain the following expression for the scattering cross section:

$$\sigma = \int_0^\infty d\omega\, \tilde{\sigma};$$

$$\tilde{\sigma}\left(\theta = \frac{\pi}{2};\ \varphi = \frac{\pi}{2}\right) = \left(\frac{e^2}{mc^2}\right)^2 \frac{1}{\Omega}\, 4\pi \left(\frac{T}{mc^2}\right)^3 A(mc)^3 F(z);$$

$$F(z) = z^2 \int_z^\infty dx\, e^{-x}\left[1 - 2\left(z^2/x^2 - z^4/x^4\right)\right]; \qquad z = \frac{mc^2}{T}\sqrt{\frac{\omega}{2\Omega}}. \tag{8}$$

The maximum of $F(z)$ occurs approximately at $z \sim 2$, which corresponds to the maximum of $\tilde{\sigma}$ over frequencies at $\omega \sim 8\left(T/mc^2\right)^2 \Omega$, i.e., the maximum is shifted far into the region of high frequencies.

The frequency shift $\omega \sim 8\left(T/mc^2\right)^2 \Omega$ is a simple consequence of the formula relating the frequencies in scattering:

$$(\omega - kv) = (\Omega - \varkappa v).$$

The physical reason for this shift is as follows: a relativistic charge scatters predominantly forward; therefore we see scattered radiation from those charges that are moving toward the observer, but the scattered radiation from these charges is shifted toward higher frequencies due to the Doppler effect.

It follows from the formula that the total cross section, integrated over all frequencies, has the value

$$\sigma\left(\theta = \frac{\pi}{2},\ \varphi = \frac{\pi}{2}\right) = \left(\frac{e^2}{mc^2}\right)^2 \left(\frac{T}{mc^2}\right)^2 \alpha,$$

where α is of order 1.

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Institute of Atomic Energy
named after I. V. Kurchatov

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Note: Figure translations are in progress. See original paper for figures.