LIGHT PROPAGATION OUTSIDE AND INSIDE THE SINGULAR SCHWARZSCHILD SPHERE

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Abstract

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LIGHT PROPAGATION OUTSIDE AND INSIDE THE SINGULAR SCHWARZSCHILD SPHERE

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PHYSICS

1. Cosmological applications of the general theory of relativity and the possibility of the existence of superdense configurations (^{1}) lead to the necessity of considering the propagation of light in a strong gravitational field possessing spherical symmetry. In investigating various processes occurring in a spherically symmetric field in vacuum, one usually uses the Schwarzschild reference system. Its applicability, as is known, is limited to the region outside the singular sphere.

However, it is possible to suppose the existence of objects lying inside their own singular sphere. An example may be a system of the Metagalaxy type, if it is spherically symmetric and has a sufficiently sharp boundary. To study the propagation of light near such objects it is necessary to turn to a reference system applicable in the world region inside the singular sphere. In this case the system is nonstatic (^{2}), and the relativistic effects can no longer be regarded as corrections to the classical solutions.

An example of reference systems including the region inside the singular sphere is the Lemaître system (^{3}), which we shall use in the present work. The interval in this system can be represented in the form (^{4})

$$ds^2 = dt^2 - \frac{1}{\left[\frac{3}{2}(x+t)\right]^{2/3}}\,dx^2 - \left[\frac{3}{2}(x+t)\right]^{4/3}(\sin^2\theta\,d\varphi^2 + d\theta^2). \qquad (1)$$

Here we have put $c = 1$, $2Gm/c^2 = 1$, where c is the fundamental velocity, G is Newton' s gravitational constant, and m is the gravitational mass of the configuration under consideration. The Lemaître system can be realized by free test particles falling from spatial infinity, where their velocity relative to the Schwarzschild reference system is zero. Thus, in the general theory of relativity, the Lemaître system realizes "parabolic motion" for each of its particles.

2. Information about processes occurring on the surface of a gravitating sphere lying inside the singular sphere can be obtained by an observer located outside the singular sphere (an external observer) only when the surface of the sphere is expanding. Otherwise, rays that have left the surface of the sphere cannot cross the singular sphere.

Let us solve the following problem: how does an observer see the expanding radiating surface of a gravitating sphere? The image of the object is formed by rays that have arrived simultaneously at the point of observation. The paths traversed by the rays are different. Therefore these rays must have been emitted by the radiating sphere at different times and, consequently, its proper dimensions at the instants of emission of the different rays are different.

However, to choose the initial conditions so that rays emerging from different points of the sphere at different instants of time arrive simultaneously at a given point of space is practically impossible. In this connection, to determine the appearance of the sphere it is expedient to solve the inverse problem, namely, to consider rays emitted simultaneously and at different angles from the point of observation, and to solve the problem of the meeting of these rays with the contracting sphere. But if the surface of the sphere is contracting while lying inside the singular sphere, then any applicable reference system (in the sense of being realizable by test particles) in this world region must necessarily be contracting, which we shall reflect in the interval (1) by replacing $t \to -t$.

Let us first solve the problem of the propagation of light rays emerging from one point in this system. The equations of light propagation and of the change of frequency ω along a ray can be written in the form (^{5}) and in our system

in polar coordinates R, θ take the form (the ray trajectory is plane)

$$\frac{d^2R}{dt^2} - \frac{0.2543}{(R-t)^{5/3}}\left(\frac{dR}{dt}\right)^3 + 1.145(R-t)^{1/3}\frac{dR}{dt}\left(\frac{d\theta}{dt}\right)^2 - \frac{0.3333}{R-t}\left(\frac{dR}{dt}\right)^2 -$$

$$-1.500(R-t)\left(\frac{d\theta}{dt}\right)^2 + \frac{0.6667}{R-t}\frac{dR}{dt} = 0,$$

$$\frac{d^2\theta}{dt^2} - \frac{0.2543}{(R-t)^{5/3}}\left(\frac{dR}{dt}\right)^2\frac{d\theta}{dt} + 1.145(R-t)^{1/3}\left(\frac{d\theta}{dt}\right)^3 + \frac{1.333}{R-t}\frac{dR}{dt}\frac{d\theta}{dt} - \frac{1.333}{R-t}\frac{d\theta}{dt} = 0, \tag{2}$$

$$\frac{1}{\omega}\frac{d\omega}{dt} - 1.145(R-t)^{1/3}\left(\frac{d\theta}{dt}\right)^2 - \frac{0.2543}{(R-t)^{5/3}}\left(\frac{dR}{dt}\right)^2 = 0.$$

To estimate the region of initial conditions under which light rays intersect the singular sphere or pass near it, an analogous problem was solved in flat space-time within the framework of the special theory of relativity, without taking into account the influence of gravitation on the propagation of light. As a result, as different variants of the initial conditions, 26 values of the angle Ψ between the ray emerging from $x = 100$* and the direction toward the center of the emitting sphere were chosen in the interval between 0° and $28°40'$.

With the initial conditions $dR/dt|_{t=0}$, $d\theta/dt|_{t=0}$, and $\omega|_{t=0} = 1$ corresponding to the angle Ψ, the quantities $R(t)$, $\theta(t)$, and $\omega(t)$ were found by integrating equations (2) by the Runge–Kutta method on the high-speed M-20 electronic computer. In the final results an accuracy of up to 1/10 of the gravitational radius was ensured. Some of the trajectories obtained are presented in Fig. 1.

The solution found was continued analytically beyond $x = 100$, where, with sufficient accuracy, the interval is represented in the form

$$ds^2 = ds_0^2 - \frac{2Gm}{c^2 r}\left(dr^2 + c^2 dt^2\right)$$

(ds_0^2 is the Galilean metric),

and thus makes it possible to study light-ray trajectories originating from any point of space.

Fig. 1. Trajectories of light rays in the Lemaître coordinate system. *a*—trajectories of rays emerging from the point $x = 100$ at the moment $t = 0$ at angles: 1—0°, 2—$1°30'$, 3—$5°15'$, 4—$6°44'$; *b*—isochrones corresponding to the position of the light-wave front at different moments (time is given in units: gravitational radius/fundamental velocity); *c*—isofrequencies (the frequency of the light emerging from $x = 100$ is taken as unity).

3. The light-ray trajectories obtained make it possible to solve various problems concerning the appearance of any body, if the motion of its emitting surface is specified. Of greatest interest for applications is the following question, whose solution is given in the present section: how will a distant external observer see a gravitating sphere whose surface is expanding with parabolic velocity.

* This value of x was chosen from the limiting considerations that, first, the singular sphere should not be too close to the observation point, and, second, the volume of computation should not increase, since for $x > 100$ an analytic continuation of the solution is possible.

In the Lemaître reference frame used, the equation of motion of this surface is $x = \text{const}$. Suppose that at some instant τ_0 the surface of the sphere begins to expand from a point. This surface, expanding over the course of $\tau_1 - \tau_0 = {}^2/_3$ grav. rad./fund., the speed of units of proper time, lies inside the singular sphere. A distant external observer will see rays of light emitted by the radiating surface during the time $\tau_0 \leq t \leq \tau_1$, when it was inside the singular sphere. The erroneousness of the widespread opinion that it is impossible to see light rays crossing the singular sphere is based on the fact that, from the point of view of the Schwarzschild reference frame, these rays left the radiating surface before the instant $t = -\infty$. Of course, this paradoxical circumstance testifies only to the inapplicability of the Schwarzschild system inside the singular sphere.

At first, after the observer records the flashed point, only rays arrive that have emerged from a small central zone of the hemisphere of the sphere facing the observer. The latter is explained by the fact that light rays emitted at large angles to the normal to the surface of the sphere cannot leave it, since the radial component of the speed of light is less than the speed of expansion of the sphere. The curvature of the trajectories of the rays under the influence of gravitation also plays a role here.

At 1.054 grav. rad./fund., the speed of units of time after the arrival of the first ray, the observer will see in the center of the visible disk rays that left the surface of the sphere at the instant when it crossed the singular sphere. At the instant these rays arrive at the observer, he sees a disk having angular dimensions $0.43\,\alpha$, where $\alpha =$ (grav. rad./phys. dist. to the singular sphere + grav. rad.)—the angular dimensions of the singular sphere. The region of the portion of the sphere visible to the observer is bounded by a small circle with polar radius 53° (instead of 90° for a static sphere in flat space), with the pole at the central point of the visible disk.

The calculation shows that if the frequency of the light emitted by the sphere is taken as unity, then at the instant under consideration the frequency of the light emitted by the central region of the visible disk will be 2.0, and by the edge 1.0. The circumstance that the frequency of the light arriving from the edge of the visible disk is less than from the central parts is explained by the fact that here the violet shift caused by the Doppler effect is stronger than at the edges of the disk. With the passage of time, the dimensions of the visible disk all increase and become greater than α.

Thus, although in the Schwarzschild reference frame (which, as already emphasized, does not cover all of space-time) the expanding sphere never had dimensions smaller than the singular sphere, nevertheless the observer sees its evolution beginning from point-like dimensions. This is also a distinctive illustration of the relativity of temporal infinity.

In conclusion, let us note that the results set forth in the present note may be used in calculations of the observed appearance of other metagalaxies or superdense configurations, as well as in problems of neutrino astronomy and cosmogony connected with neutrino ejections during explosions of superdense bodies.

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Note: Figure translations are in progress. See original paper for figures.

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