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Abstract

Full Text

THEORY OF ELASTICITY

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ON THE LOWER CRITICAL LOAD FOR CYLINDRICAL SHELLS

In the author's papers (1–3), postcritical deformations of cylindrical shells under various modes of loading—axial compression, external pressure, and torsion—were considered. These considerations applied to shells of unlimited elasticity. For real shells, which possess limited elasticity, the results obtained there are applicable only in the case of sufficiently thin shells.

Thus, for example, for the lower critical load under axial compression of a cylindrical shell, the formula

$$p_i = 0.18E\frac{\delta}{R},$$

was obtained, where R is the radius, δ is the shell thickness, and E is the elastic modulus of the material. For shells with limited elasticity of the material, this formula is applicable only when the condition

$$2.5E\frac{\delta}{R} < \sigma_{\mathrm{B}},$$

is satisfied, where σ_{B} is the ultimate strength. (It is assumed that the shell material has the classical stress–strain diagram with a horizontal segment throughout the entire range of plastic deformations.)

In the present note we consider comparatively thick shells. These are shells in which the postcritical deformations associated with transition to a stable equilibrium state after "snap-through" cause stresses to appear beyond the elastic limit of the material. In particular, for cylindrical shells operating under conditions of axial compression, comparatively thick shells will be those for which

$$\sigma_{\mathrm{B}} < 2.5E\frac{\delta}{R}.$$

As shown in papers (1–3), postcritical deformations of cylindrical shells in all principal cases of loading, after loss of stability, are accompanied by the formation of sharply pronounced ridges on the surface, along which considerable bending stresses arise. It turns out that, for shells with limited elasticity, the ridge form in which elastic-plastic deformation from local bending arises is energetically unfavorable, since this arrests the deformation (4). Therefore, in all loading cases for comparatively thick shells we require that, in the process of postcritical deformation, the stresses from local bending along the ridges not exceed the ultimate strength of the material σ_B.

Satisfaction of the indicated condition leads to an increase in the lower critical load. In particular, *for the value of the lower critical load under axial compression of the shell*, the formula obtained is

$$p_i = \bar{p}_i E \frac{\delta}{R},$$

where

$$\bar{p}_i = 0.03 \left\{ 0.6 - 2\omega + \left(\sqrt{0.5 + \frac{1}{\omega^3}} + \sqrt{2\omega + 0.5 + \frac{1}{\omega^3}} \right)^2 \right\}, \quad \omega = 0.6 \frac{\sigma_\text{B} R}{E\delta}.$$

In view of the condition $\sigma_\text{B} < 2.5 E\delta/R$, this formula applies to shells for which $\omega < 1.5$. It should be noted, however, that the formula cannot be used for shells with a value of the parameter ω appreciably less than unity.

The lower critical load for a cylindrical shell under external pressure, in the case of unlimited elasticity of the material, is determined by the formula

$$q_i = \bar{q}_i E \left(\frac{\delta}{R} \right)^2,$$

where

$$\bar{q}_i = 2.26 \left(\frac{R\delta}{l^2} \right) \left\{ \left(\frac{R\delta}{l^2} \right)^{-1/6} + 0.6 \right\}.$$

This formula is also applicable to sufficiently thin shells with limited elasticity, if

$$0.37 E \frac{\delta}{R} \left(\frac{\delta R}{l^2} \right)^{-1/3} \leq \sigma_\text{B}.$$

Relatively thick shells are defined by us as those for which

$$\sigma_\text{B} < 0.37 E \frac{\delta}{R} \left(\frac{\delta R}{l^2} \right)^{-1/3}.$$

For them

$$\bar{q}_i = \left(\frac{R\delta}{l^2}\right)\left\{\left(\frac{0.75}{\omega^2} + 1.5\omega\right)\left(\frac{R\delta}{l^2}\right)^{-1/6} + 1.33\right\},$$

where

$$\omega = 0.37\frac{E\delta}{\sigma_B R}\left(\frac{R\delta}{l^2}\right)^{-1/3}.$$

This formula may be used if

$$\frac{E\delta}{\sigma_B R} \leq 1.$$

For the lower critical load in torsion of cylindrical shells, in the case of unlimited elasticity of the material, the formula obtained is

$$s_i = \bar{s}_i E\frac{\delta}{R}, \qquad \bar{s}_i = 0.22\left(\frac{R\delta}{l^2}\right)^{1/4}.$$

This formula is applicable to thin shells with limited elasticity, provided that the condition

$$3.5E\frac{\delta}{R} < \sigma_B$$

is satisfied.

In the case of relatively thick shells, i.e., shells satisfying the condition

$$\sigma_B < 3.5E\frac{\delta}{R},$$

$$\bar{s}_i = 0.0272\left(2 + \omega + \sqrt{\omega\left(\omega + 4.4\sqrt{a}\right)}\right),$$

where

$$\sqrt{a} = \frac{1}{1.8}\frac{\sigma_B R}{E\delta}, \qquad \omega = 1 + 4.4a^{-3/2},$$

the formula cannot be used for excessively small values of the parameter a.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.