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GAS MOTION BEHIND AN ASYMMETRIC PISTON

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Abstract

Full Text

HYDROMECHANICS

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GAS MOTION BEHIND AN ASYMMETRIC PISTON

(Presented by Academician L. I. Sedov on 5 VII 1962)

Let us consider the motion of a piston that moves according to the law $r = ut[1 + \varepsilon f(\theta)]$, where u is a constant quantity; ε is a small constant quantity; t is time; r, θ are polar coordinates, if the piston is cylindrical, and spherical coordinates, if the piston is axisymmetric.

We assume that the gas in front of the piston is at rest, its density is ρ_1 , and its pressure is p_1 . Instead of the coordinate r we introduce the coordinate r_1 ; r and r_1 are related by $r = r_1[1 + \varepsilon f(\theta)]$. All the unknown functions depend on r_1, θ, t , and the coordinates r_1, θ are not orthogonal. The basic equations of gas dynamics in the coordinates r_1, θ will have the form

$$[1 + \varepsilon f(\theta)] \frac{\partial \rho}{\partial t} + \frac{\partial \rho v_{r_1}}{\partial r_1} + \frac{1}{r_1} \frac{\partial \rho v_\theta}{\partial \theta} + \frac{\rho}{r_1} [(\nu - 1)v_{r_1} + (\nu - 2)v_\theta \operatorname{ctg} \theta + (\nu - 1)\varepsilon f'(\theta)] = 0,$$

$$[1 + \varepsilon f(\theta)] \frac{\partial}{\partial t} \left(\frac{p}{\rho^\gamma} \right) + v_{r_1} \frac{\partial}{\partial r_1} \left(\frac{p}{\rho^\gamma} \right) + \frac{v_\theta}{r_1} \frac{\partial}{\partial \theta} \left(\frac{p}{\rho^\gamma} \right) = 0,$$

$$[1 + \varepsilon f(\theta)] \frac{\partial v_{r_1}}{\partial t} + v_{r_1} \frac{\partial v_{r_1}}{\partial r_1} + \frac{v_\theta}{r_1} \frac{\partial v_{r_1}}{\partial \theta} - \frac{v_\theta^2}{r_1} [1 + \varepsilon f(\theta)] + \varepsilon f'(\theta) \left(\frac{\partial v_\theta}{\partial t} + v_{r_1} \frac{\partial v_\theta}{\partial r_1} + \frac{v_\theta}{r_1} \frac{\partial v_\theta}{\partial \theta} \right) = -\frac{1}{\rho} \frac{\partial p}{\partial r_1}, \tag{1}$$

$$[1 + \varepsilon f(\theta)] \frac{\partial v_\theta}{\partial t} + v_{r_1} \frac{\partial v_\theta}{\partial r_1} + \frac{v_\theta}{r_1} \frac{\partial v_\theta}{\partial \theta} + \frac{v_{r_1} v_\theta}{r_1} + \varepsilon f'(\theta) \left(\frac{\partial v_{r_1}}{\partial t} + v_{r_1} \frac{\partial v_{r_1}}{\partial r_1} + \frac{v_\theta}{r_1} \frac{\partial v_{r_1}}{\partial \theta} \right) = -\frac{1}{r_1 \rho} \frac{\partial p}{\partial \theta},$$

where ρ is density; p is pressure; v_{r_1}, v_θ are the velocity components; $\nu = 2; 3$, respectively, for a cylindrical and an axisymmetric piston; $\gamma = c_p/c_v$. Introduce the dimensionless quantity $\lambda = \frac{\gamma p_1 t^2}{\rho r_1^2}$, and represent the unknown functions through dimensionless functions depending on the dimensionless variables λ, θ :

$$v_{r_1} = \frac{r_1}{t} \bar{V}_{r_1}(\lambda, \theta), \quad v_\theta = \frac{r_1}{t} \bar{V}_\theta(\lambda, \theta),$$

$$p = \rho_1 \left(\frac{r_1}{t}\right)^2 \bar{P}(\lambda, \theta), \quad \rho = \rho_1 \bar{R}(\lambda, \theta). \quad (2)$$

We shall seek solutions for $\bar{V}_{r_1}, \bar{V}_\theta, \bar{P}, \bar{R}$ in the form

$$\bar{V}_{r_1} = V_0 + \varepsilon f(\theta) V_{r_1}(\lambda), \quad \bar{V}_\theta = -\varepsilon f'(\theta) V_\theta(\lambda),$$

$$\bar{P} = P_0 + \varepsilon f(\theta) P(\lambda), \quad \bar{R} = R_0 + \varepsilon f(\theta) R(\lambda), \quad (3)$$

where V_0, P_0, R_0 are the solutions for the self-similar motion of gas behind a piston moving with constant velocity in a homogeneous medium, obtained by L. I. Sedov ⁽¹⁾. The function $f(\theta)$ for a cylindrical piston must satisfy the equation

$$f''(\theta) + k_1 f(\theta) = 0. \quad (4)$$

For an axisymmetric piston, $f(\theta)$ must satisfy Legendre's equation

$$f''(\theta) + \operatorname{ctg} \theta f'(\theta) + k_2 f(\theta) = 0. \quad (5)$$

For the functions V_{r_1}, V_θ, P, R one obtains a system of linear nonhomogeneous equations, the solution of which is composed of the general solution of the homogeneous system and a particular solution of the nonhomogeneous system of equations. Passing from the independent variable λ to the independent variable V_0 , we seek the general solution of the homogeneous system in the form:

Fig. 1 Fig. 2

Fig. 1

Fig. 2

$$V_{r_1} = (1 - V_0)^s \sum_{n=0}^{\infty} a_{ni} (1 - V_0)^n, \quad V_\theta = (1 - V_0)^s \sum_{n=0}^{\infty} d_{ni} (1 - V_0)^n,$$

$$P = (1 - V_0)^s \sum_{n=0}^{\infty} b_{ni} (1 - V_0)^n, \quad R = (1 - V_0)^s \sum_{n=0}^{\infty} c_{ni} (1 - V_0)^n, \quad (6)$$

where λ, P_0, R_0 are also represented in the form of expansions in powers

$$\lambda = \lambda_n \left[1 - \frac{2}{\nu} (1 - V_0) \dots \right], \quad P_0 = P_{0n} \left[1 - \frac{2}{\nu} (1 - V_0) \dots \right], \quad (7)$$

Fig. 3

Figure 1: Fig. 3

$$R_0 = R_{0n} [1 + \alpha(1 - V_0)^2 \dots],$$

where $\lambda_n, P_{0n}, R_{0n}$ are the values of the functions on the piston.

The characteristic equation of the homogeneous system will be

$$S^3 \left(S - \frac{1}{\nu} \right) = 0. \quad (8)$$

The roots $S_1 = S_2 = S_3 = 0$ correspond to a logarithmic singularity.

We shall also seek the particular solution of the nonhomogeneous system in the form of a series in powers of $(1 - V_0)$.

Let us consider the boundary conditions. On the piston $v_n = u[1 + \varepsilon f(\theta)]$, and since $V_0 = 1$, it follows that $V_{r1} = 1$.

We shall assume that the piston velocity is large, and take the conditions on the shock wave in the form

$$v_2 = \frac{2}{\gamma + 1}c, \quad \rho_2 = \frac{\gamma + 1}{\gamma - 1}\rho_1, \quad p_2 = \frac{2}{\gamma + 1}\rho_1c^2, \quad (9)$$

where c is the shock-wave velocity.

The conditions on the shock wave are obtained in the form

$$V_{r2} = \frac{2}{\gamma + 1}, \quad V_{\theta 2} = \frac{2}{\gamma + 1}[1 + a], \quad P_2 = \frac{4}{\gamma + 1}, \quad R_2 = 0, \quad (10)$$

where a is a constant quantity if r_2 at the shock wave is represented by the formula

$$r_2 = r_{20}[1 + a\varepsilon f(\theta)]. \quad (11)$$

Let $f(\theta) = \cos \theta$; in this case equations (4) and (5) will be satisfied, i.e., the motion can be considered as that of either a cylindrical or an axisymmetric piston.

Fig. 3

The final formulas for V_{r1}, V_{θ}, P, R , if $(1 - V_0)^2$ is neglected, take the form:

cylindrical piston, i.e. $\nu = 2$:

$$\begin{aligned}
 V_{r1} &= 1 - (1 - V_0) \left\{ 1 + \frac{b_{01}}{2R_{0n}} [8 + 3 \ln(1 - V_0) + \ln^2(1 - V_0)] + \frac{d_{02}}{3} \sqrt{1 - V_0} \right\}, \\
 V_\theta &= \frac{b_{01}}{R_{0n}} \left\{ [1 + \ln(1 - V_0) + \ln^2(1 - V_0)] V_0 + 2[5 + 2 \ln(1 - V_0)] + \right. \\
 &\quad \left. + [7 - 6 \ln(1 - V_0)](1 - V_0) \right\} + d_{02} \sqrt{1 - V_0} \left[1 - \frac{1}{2}(1 - V_0) \right] - (1 - V_0), \\
 P &= b_{01} \left\{ [1 + \ln(1 - V_0) + \ln^2(1 - V_0)] V_0 + 3(1 - V_0) [2 \ln(1 - V_0) - 1] \right\}, \quad R = 0; \\
 &\hspace{15em} (12)
 \end{aligned}$$

axisymmetric piston, i.e. $\nu = 3$:

$$\begin{aligned}
 V_{r1} &= 1 - (1 - V_0) \left\{ 1 + \frac{2}{3} \frac{b_{01}}{R_{0n}} [17 + 5 \ln(1 - V_0) + \ln^2(1 - V_0)] + \frac{d_{02}}{2} \sqrt{1 - V_0} \right\}, \\
 V_\theta &= \frac{b_{01}}{R_{0n}} \left\{ [1 + \ln(1 - V_0) + \ln^2(1 - V_0)] \left[1 - \frac{2}{3}(1 - V_0) \right] + \right. \\
 &\quad \left. + 3[7 + 2 \ln(1 - V_0)] - (1 - V_0) [4 + 3 \ln(1 - V_0)] \right\} + \\
 &\quad + d_{02} \sqrt[3]{1 - V_0} \left[1 - \frac{1}{3}(1 - V_0) \right] - (1 - V_0), \\
 &\hspace{15em} (13)
 \end{aligned}$$

$$P = b_{01} \left\{ 1 + \ln(1 - V_0) + \ln^2(1 - V_0) - \frac{2}{3}(1 - V_0) [4 - 5 \ln(1 - V_0) + \ln^2(1 - V_0)] \right\}, \quad R = 0.$$

Using these formulas, V_{r1}, V_θ, P, R have been calculated and are presented in the form of graphs (see Figs. 1, 2, 3).

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CITED LITERATURE

1. L. I. Sedov, *Similarity and Dimensional Methods in Mechanics*, Moscow, 1957.
2. G. Piaggio, *Integration of Differential Equations*, 1933.

Note: Figure translations are in progress. See original paper for figures.

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