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Abstract

Full Text

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AN INTEGRAL LIMIT THEOREM FOR LARGE DEVIATIONS

(Presented by Academician A. N. Kolmogorov on 7 VII 1962)

Let $\mathfrak{G}$ be the class of distribution functions $G(x)$, absolutely continuous for $x > B$ (B depends on $G(x)$), with a density that can be represented in the form

$$G'(x) = \int_0^\infty e^{-xu}\varphi(u)\,du,$$

where $\varphi(u) \geq 0$ is integrable, has a second derivative satisfying the Hölder condition, and $\varphi'(0) = 0$. It is evident that distributions from the class $\mathfrak{G}$ do not satisfy Cramér' s well-known condition ([1]).

Theorem. Let ξ_i be a sequence of independent identically distributed random variables with distribution function $F(x)$, where $M\xi_i = 0$, and let $F_n(x)$ be the distribution function of the sum $\xi_1 + \xi_2 + \cdots + \xi_n$. If there exists $G(x) \in \mathfrak{G}$ such that, for any $b > a > B$,

$$\operatorname*{Var}_{a \le x \le b}[F(x) - G(x)] \leq \int_a^b \frac{\psi(x)}{x^2}\,G'(x)\,dx, \tag{1}$$

where $\psi(x)$ is integrable on (B, ∞), then

$$1 - F_n(x) = n(1 - F(x))(1 + O(1))* \tag{2}$$

for such x that

$$nu_x^2 = O(1), \tag{3}$$

where u_x is the solution of the equation

$$u_x e^{xu_x}(1 - G(x)) = 1.$$

In particular, if $\varphi(u) \sim u^\alpha,\ \alpha > 2$, as $u \to 0 * *$, then (2) holds for

$$x > \sqrt{n}\ln n\,\rho(n),$$

where $\rho(n)$ is an arbitrary function such that

$$\lim_{n\to\infty} \rho(n) = \infty.$$

Recently V. V. Petrov ([4]) obtained, for $0 \leq x \leq n^{\alpha}/\rho(n)$, where $\alpha < 1/2$, an asymptotic expression for $1 - F_n(x)$ of the same type as in Cramér' s work ([1]), under the assumption that

$$M \exp |\xi_1|^{\frac{4\alpha}{2\alpha+1}} < \infty. \tag{4}$$

If, along with (4), condition (2) is also satisfied, then simple calculations show that the x satisfying (3) grow faster than n^{α}.

The question of large deviations for intermediate values of x remains open.

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REFERENCES

1. H. Cramer, *Actuel sci. et ind.*, No. 736 (1938).

2. S. V. Nagaev, *Vestn. LGU*, No. 1 (1962).

3. Yu. V. Linnik, *Proc. of the Fourth Berkeley Symposium on Mathem. Statistics and Probability*, 2, 1961.

4. V. V. Petrov, *DAN*, 138, No. 4 (1961).

* An analogous representation is obtained in ([3]) under the condition that

$$1 - F(x) = \frac{A_a}{x^a} + \frac{A_{a+1}}{x^{a+1}} + \cdots + \frac{A_{4a+5}}{x^{4a+5}} + O\left(\frac{1}{x^{4a+5+\varepsilon}}\right),$$

where $a \geq 3$ is an integer, and A_i are constants.

** Under the same qualitative condition, local limit theorems were proved in ([2]).