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Abstract

Full Text

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ANALYTIC PROPERTIES OF THE MESON-NUCLEON VERTEX PART AND OF THE SCATTERING AMPLITUDE OF PSEUDOSCALAR MESONS IN PERTURBATION THEORY

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For establishing dispersion relations and spectral representations of matrix elements in any finite order of perturbation theory, a method of majorizing Feynman diagrams was developed in papers ⁽¹⁻⁵⁾. This method makes it possible to find a finite number of strongly connected diagrams—**primitive diagrams**—which determine the domain of analyticity of all strongly connected diagrams of the given process in the space of Euclidean external momenta. However, in the case of scattering of a pseudoscalar meson by a meson (or by a nucleon), as well as in the case of the π -meson-nucleon vertex part, the primitive diagrams found in ⁽²⁻⁴⁾ are not physical, since they contain vertices at which three meson lines meet.

The present note is devoted to the majorization of diagrams with allowance for the pseudoscalarity of π -mesons. Physical primitive diagrams are found for the meson-nucleon vertex part and for the amplitude of meson scattering by a meson, and the corresponding domains of analyticity are determined.

Consider the class R of all strongly connected diagrams of some process involving π -mesons and nucleons. We shall assume that at each vertex of a diagram $D \in R$ either two nucleon lines and one meson line meet, or four meson lines meet*. We shall also assume that the diagrams of the class R contain no closed nucleon paths—cycles consisting of an odd number of nucleon lines (owing to the invariance of the S -matrix in the theory of strong interactions with respect to space reflections, the total contribution from diagrams with odd cycles is equal to zero). Our aim will be to find the domain

$$G_R = \bigcap_{D \in R} G(D),$$

where $G(D)$ is the domain in the space of Euclidean external momenta in which the quadratic form of the diagram $D \in R$

Fig. 1. Circles are external vertices; heavy lines are nucleon lines, thin lines are π -meson lines

Figure 1: Fig. 1. Circles are external vertices; heavy lines are nucleon lines, thin lines are π -meson lines

$$Q(\alpha, p) = \sum_{i,j} A_{ij}(\alpha) p_i p_j - \sum_{\nu} \alpha_{\nu} m_{\nu}^2 \quad (1)$$

is negative ⁽²⁻⁴⁾ (here p_i are external momenta, m_{ν} are the masses on the internal lines of the diagram, and $\alpha_{\nu} \geq 0$ are Feynman parameters).

Lemma 1. *The class R is majorized by its subset R_1 , consisting of those diagrams from R which contain no nucleon cycles: $G_R = G_{R_1}$.*

Proof. Let a diagram $D \in R$ contain a nucleon cycle with vertices $1, 2, \dots, 2n - 1, 2n$. Replace each of the nucleon lines $(1, 2), (3, 4), \dots, (2n - 1, 2n)$ by a pair of π -meson lines, and the lines $(2, 3), (4, 5), \dots, (2n, 1)$ by one meson line. By Symanzik's theorem ^(1,3,4), the diagram obtained in this way majorizes the diagram D , since $2m < M$ (M is the nucleon mass, m the π -meson mass). Repeating the operation described for each nucleon cycle of the diagram D , we see that any diagram from R is majorized by some diagram from R_1 .

* By decreasing the masses on internal lines, one can show (see ⁽²⁾) that a broader class of diagrams, some internal lines of which correspond to strange particles, reduces to this class.

Lemma 1 makes it possible to restrict our consideration to the class R_1 , each diagram of which may contain nucleon lines only in the form of (unclosed) broken lines—nucleon polygons. Using the methods set out in ⁽⁴⁾, § 3 (see Theorems 1 and 2 and Lemmas 3 and 4 of ⁽⁴⁾*), one can show that all diagrams of the class R_1 are majorized by a finite set of diagrams $R_2 \subset R_1$, i.e. that $G_{R_1} = G_{R_2}$.

Theorem 4.1: “every diagram is majorized by any of its subdiagrams” plays the fundamental role in the majorization process. Its proof, given in ⁽²⁾, must be supplemented (with the class of diagrams R_1 in mind) by the proof of the following assertion. If a diagram $D \in R_1$ is obtained from a diagram $D_1 \in R_1$ by removing an internal meson vertex (i.e. a vertex at which four meson lines meet) and by “separating” two meson lines, then D majorizes D_1 . This assertion is a simple consequence of Symanzik's theorem, which has been noted in the literature (see ⁽¹⁾, Fig. 1e, and also ⁽⁵⁾, Lemma 2). Using it, one can get rid of all diagrams of the class R_1 that contain internal meson vertices.

Fig. 1. Circles are external vertices; heavy lines are nucleon lines, thin lines are π -meson lines

Next, using Symanzik's theorem and its generalization^(3,4), and by directly investigating the quadratic forms of the diagrams from R_2 , one can show that some of the diagrams in the set R_2 are majorized by the remaining diagrams of this set. Thus we arrive at the class of primitive diagrams R_0 ($R_0 \subset R_2 \subset R_1 \subset R$), which majorize all strongly connected diagrams of the given process: $G_R = G_{R_0}$.

Using the described method of majorizing diagrams, we shall prove the following two theorems.

Theorem 1. *All strongly connected diagrams of the π -meson-nucleon vertex part are majorized by the diagrams D_1 and D_2 of Fig. 1.*

Proof (we list only the main stages of the argument). Let 1 and 2 be the external nucleon vertices, and 3 the external meson vertex of an arbitrary diagram $D \in R_1$. Two cases are possible, depending on whether the vertex 3 lies on the nucleon polygon or three internal meson lines meet at the vertex 3. In the first case, with the aid of Lemmas 4.3 and 4.4 and Theorem 4.1, it is not difficult to prove that the diagram D is majorized by one of the two diagrams D_1 or D_3 in Fig. 1. In the second case, combining the condition of strong connectedness (Lemma 4.3) and the condition of pseudoscalarity of the π -mesons (expressed in the fact that four lines meet at each meson vertex), we come to the conclusion that in the diagram D there exist three meson paths (with no common line sections) connecting the vertex 3 with the nucleon polygon. After this, by simple combinatorics and by successive application of Theorems 4.2 and 4.1, we find that the diagram D is majorized by one of the diagrams D_2 , D_4 , or D_5 in Fig. 1. Thus, the class R_2 , in the case of the meson-nucleon vertex part, consists of the five diagrams in Fig. 1.

Next, using Symanzik's theorem, one can prove that the diagram D_3 is majorized by the diagram D_1 , and the diagram D_5 by the diagram D_4 . Finally, the diagram D_4 is majorized by the two diagrams D_1 and D_2 (i.e. $G(D_4) \supset G(D_1) \cap G(D_2)$) by virtue of the following lemma.

Lemma 2. *If in the diagram D' there is a part shown in Fig. 2a, then the Euclidean domain of analyticity of this diagram $G(D')$*

* In what follows we shall refer to these results as Theorems 4.1 and 4.2 and Lemmas 4.3 and 4.4.

contains the intersection of the regions $p_3^2 < 9m^2$ and $G(D)$, where D is the diagram obtained from D' by replacing the part shown in Fig. 2a by the part shown in Fig. 2b.

This lemma is proved analogously to Lemma 4 of work⁽³⁾. It is also useful in the study of diagrams for the scattering of a π -meson by a nucleon.

It follows from Lemma 2 that the diagram D_4 is majorized by the diagram D_1 , if $p_3^2 < 9m^2$. But in the region $G(D_2)$ this inequality is satisfied automatically.

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

This completes the proof of Theorem 1.

It can be shown that Theorem 1 does not admit refinement in the sense that neither of the diagrams D_1 and D_2 in Fig. 1 majorizes the other.

Fig. 2

Corollary. If $p_1^2 = p_2^2 = M^2$, then in any order of perturbation theory the vertex function is analytic with respect to $z = p_3^2$ in the complex z -plane, with a cut along the real axis from $9m^2$ to ∞ .

Proof. In the Euclidean region (i.e., in the interval $0 \leq p_3^2 \leq 4M^2$), according to Theorem 1, the nearest singularity is given by the diagram D_2 and is equal to $p_3^2 = 9m^2$ (since $3m < 2M$). This result can be used to prove analyticity in the cut z -plane, if one also notes that the quadratic form Q (1) of an arbitrary diagram D of the vertex part can be written in the form ⁽¹⁰⁾

$$Q(\alpha, p) = A_1(\alpha)p_1^2 + A_2(\alpha)p_2^2 + A_3(\alpha)p_3^2 - \sum_{\nu} \alpha_{\nu} m_{\nu}^2, \quad (2)$$

where

$$A_i(\alpha) \geq 0 \quad \text{for all } \alpha_{\nu} \geq 0, \quad (3)$$

and that the contribution from the diagram D to the vertex part can have a singularity only at points where $Q(\alpha, p) = 0$.

Theorem 2. All strongly connected diagrams for the scattering of a π -meson by a π -meson are majorized by the diagram D of Fig. 3 with different numberings of the external momenta.

We shall not give here the very cumbersome proof of this theorem. We note only that in the case of π - π scattering the class R_1 consists of diagrams containing only π -meson lines; therefore, in majorizing it is sufficient to use Theorem 1 from ⁽⁴⁾ and Lemma 3 from ⁽⁴⁾. The class R_2 in the present case consists of the two diagrams shown in Fig. 3. The diagram D' is majorized by two diagrams of type D (with different numberings of the external vertices) by virtue of the generalized Symanzik theorem.

Fig. 3

Corollary. If the external meson momenta lie on the mass surface (i.e., $p_i^2 = m^2$), then all strongly connected diagrams of the π - π -scattering process are analytic in the triangle*

$$s = (p_1 + p_2)^2 < 4m^2, \quad t = (p_1 + p_3)^2 < 4m^2, \quad u = (p_1 + p_4)^2 < 4m^2 \quad (4)$$

on the plane $s + t + u = 4m^2$.

The proof of this assertion is based on the study of six diagrams of type D (Fig. 3) and on the decomposition of the real vectors p into mutually orthogonal Euclidean and anti-Euclidean parts**. In doing so it is necessary

* We assume that all external lines are incoming, so that the conservation law for four-momentum is written here in the form $p_1 + p_2 + p_3 + p_4 = 0$.

** We note that the analytic properties of diagrams of type D were studied directly (without passing to majorizing Euclidean vectors) in works ^(8,9).

one should bear in mind that the matrix \bar{A}_{ij} entering expression (1), for the quadratic form $Q(\alpha, p)$, is positive definite, and that the majorization has been carried out not only on the mass shell, but also for arbitrary (variable) squares of the external momenta*.

From analyticity in the real domain (4), taking into account the linearity of the form Q with respect to the invariants s and t , in (6) a certain complex domain of analyticity has been obtained in any order of perturbation theory. The established analytic properties of the π - π -scattering amplitude are sufficient for proving one-dimensional dispersion relations both at fixed t ($-4m^2 < t < 4m^2$) and at fixed s ($-4m^2 < s < 4m^2$). They also make it possible to prove dispersion relations for the partial waves of π - π scattering (see, for example, (7)).

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REFERENCES

1. K. Symanzik, *Progr. Theor. Phys.*, **20**, 960 (1958).
2. A. A. Logunov, A. N. Tavkhelidze and L. D. Landau, **135**, 801 (1960).
3. A. A. Logunov, I. T. Todorov, N. A. Chernikov, *Problems of the Theory of Majorization of Feynman Diagrams*, Preprint of the Joint Institute for

Nuclear Research, D-578, Dubna, 1960.

4. A. A. Logunov, I. T. Todorov, N. A. Chernikov, ZhETF, **42**, 1285 (1962).
5. I. A. Drabkin, Yu. A. Yappa, Vestn. Leningrad. Univ., No. 4, 28 (1962).
6. T. T. Wu, Phys. Rev., **123**, 678 (1961).
7. J. G. Taylor, Nuovo Cim., **22**, 92 (1961).
8. S. Mandelstam, Phys. Rev., **115**, 1752 (1959).
9. V. A. Kolkunov, L. B. Okun' et al., ZhETF, **39**, 340 (1960).
10. N. Nakanishi, Suppl. Progr. Theor. Phys., No. 18, 1 (1961).

* See also (6), where an essentially equivalent derivation is given for the analyticity of the contribution from diagrams in the domain (4) in the case of scalar-meson scattering. The derivation outlined here has the advantage that it carries over without change to the case of unequal masses.

Note: Figure translations are in progress. See original paper for figures.

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