# Physics

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### Abstract

### Full Text

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# ON THE ANOMALOUS DEMBER EFFECT

**1.** According to the theory of the Dember effect ([1,2]), the sign of the potential difference arising between the illuminated and the shadow faces of a semiconductor coincides with the sign of the charge of the less mobile carriers. This result appears self-evident, since the effect itself is caused by the fact that, in the process of photodiffusion, the more mobile carriers outstrip the less mobile ones. It turns out, however, that under certain conditions an anomalous Dember effect can arise, possessing the opposite sign of the photo-emf. The fundamental possibility of such an inverse photodiffusion effect follows from the qualitative considerations of Kallmann, Kramer, Heidenakis, Macalear, Barkenmeier, and Polak ([3]), concerning the interpretation of the properties of high-voltage photo-emf in semiconductor films. Examination shows that the loss of this result in the theory of the Dember effect is connected with particular assumptions under which the corresponding kinetic equations were solved. In the present article a development of the theory of the Dember effect is given.

**2.** In order to write explicitly the photodiffusion emf

$$\mathcal{E} = \frac{kT}{q}\int_0^l \frac{dp - b\,dn}{p + bn}, \tag{1}$$

one must solve the system of kinetic equations ($p' = p - p_0$; $n' = n - n_0$; $\lambda, \lambda_1$ are the coefficients of total and photoactive absorption of light; $I$ is the intensity of light; $G$ is the rate of surface photogeneration of pairs; $L = \sqrt{D\tau}$ is the diffusion length; $0, l$ are the coordinates of the illuminated and shadow faces of the semiconductor; the remaining notation is generally accepted):

$$\frac{\partial p'}{\partial x} - \frac{qE}{kT}(p' + p_0) + \frac{i_p}{qD} = 0;$$

$$\frac{\partial \rho}{\partial x} + \frac{qE}{kT}\rho - \frac{q^2E}{kT}(2p' + p_0 + n_0) + \frac{b-1}{bD}i_p = 0;$$

$$\frac{1}{q}\frac{\partial i_p}{\partial x} + \frac{p'}{\tau}\frac{\partial p'}{\partial t} - \lambda_1 I e^{-\lambda x} = 0; \tag{2}$$

$$\frac{\partial i}{\partial x} + \frac{\partial \rho}{\partial t} = 0;$$

$$\frac{\partial E}{\partial x} - \frac{4\pi}{\varepsilon}\rho = 0;$$

$$n' = p' - \frac{\rho}{q};$$

$$i_n = -i_p.$$

The nonlinearity of this system is due to the expressions $p'E$, $\rho E$, and also to the concentration dependence of the lifetime $\tau = \tau(p_0, n_0, p', p)$ in re-

recombination term $p'/\tau$. We shall seek the solution in the following form $(l \gg L'_\mathrm{D}, L''_\mathrm{D})$:

$$p' \approx \mathrm{const}; \qquad i_p \approx \mathrm{const} \quad \text{for } x \lesssim 3L'_\mathrm{D}, \quad |l - x| \lesssim 3L''_\mathrm{D};$$

$$\rho \approx 0; \qquad E \approx \mathrm{const} \quad \text{for } 3L'_\mathrm{D} \lesssim x \lesssim l - 3L''_\mathrm{D}, \tag{3}$$

where

$$L_\mathrm{D} = \sqrt{\frac{\varepsilon kT}{4\pi q^2(n_0 + p_0)}}; \qquad L'_\mathrm{D} = \sqrt{\frac{\varepsilon kT}{4\pi q^2(n_0 + p_0 + 2p'_0)}};$$

$$L''_\mathrm{D} = \sqrt{\frac{\varepsilon kT}{4\pi q^2(n_0 + p_0 + 2p'_l)}}. \tag{4}$$

From the dimensionless form of the equations for $\rho$ and $E$ in the boundary layers (the units of measurement of $x$, concentrations, $\rho$, $E$, and $i$ are respectively: $\bar{x} = L'_\mathrm{D}$ or $L''_\mathrm{D}$; $\bar{n} = n_0 + p_0 + 2p'_0$ or $n_0 + p_0 + 2p'_l$; $\bar{\rho} = q\bar{n}$; $\bar{E} = kT/q\bar{x}$; $\bar{i} = qD\bar{n}/\bar{x}$)

$$\frac{\partial \rho}{\partial x} - E + \frac{b-1}{b} i_p + \rho E = 0;$$

$$\frac{\partial E}{\partial x} - \rho = 0, \tag{5}$$

it follows that for all real values of the illumination the quadratic term $\rho E$ is negligibly small, and the solution has the following form:

for $0 \lesssim x \lesssim 3L'_\mathrm{D}$

$$\rho = \frac{\varepsilon}{4\pi}\frac{A_1}{L'_\mathrm{D}} e^{(l-x)/L'_\mathrm{D}}; \qquad E = B_1 - A_1 e^{-x/L'_\mathrm{D}}, \tag{6}$$

for $l - 3L''_\mathrm{D} \lesssim x \lesssim l$

$$\rho = -\frac{\varepsilon}{4\pi}\frac{A_2}{L''_\mathrm{D}} e^{x/L''_\mathrm{D}}; \qquad E = B_2 - A_2 e^{-(l-x)/L''_\mathrm{D}}.$$

Solving the quasineutral problem in the bulk of the material, we find

$$p' = B_3 e^{-\lambda x} + A_3 e^{\gamma_1 x} + A_4 e^{\gamma_2 x};$$

$$i_p = q\mu p_0 \widetilde{E} + B_4 e^{-\lambda x} - \frac{q}{\tau^* \gamma_1} A_3 e^{\gamma_1 x} - \frac{q}{\tau^* \gamma_2} A_4 e^{\gamma_2 x}. \tag{7}$$

Here

$$\gamma_{1,2} = \frac{q\widetilde{E}}{2kT} \mp \sqrt{\left(\frac{q\widetilde{E}}{2kT}\right)^2 + \frac{1}{D\tau} + \frac{s}{D}};$$

$$B_1 = \frac{(b-1)i_{p0}}{qb\mu\,(n_0 + p_0 + 2p'_0)};$$

$$B_2 = \frac{(b-1)i_{pl}}{qb\mu\,(n_0 + p_0 + 2p'_l)}; \tag{8}$$

$$B_3 = \frac{\lambda_1 I\tau}{1 - \lambda(\lambda + q\widetilde{E}/kT)D\tau + s\tau};$$

$$B_4 = \frac{q\lambda_1 I D\tau(\lambda + q\widetilde{E}/kT)}{1 - \lambda(\lambda + q\widetilde{E}/kT)D\tau + s\tau};$$

$$\tau^* = \frac{\tau}{1 + s\tau}.$$

Solution (7) was obtained for $\tau = \mathrm{const}$, i.e., according to (4), it is valid when

small and large generation levels. The expressions $p(s)$, $\widetilde{E}(s)$, $p'(s)$, and $i_p(s)$ are the transforms of the corresponding quantities on the basis of the Laplace transformation (5). $\widetilde{E}(s)$ in expressions (7) and (8) is quasiconstant, i.e., a slow function of the coordinate.

From the boundary conditions:

at $x = 0$

$$E = 0; \qquad G - \varkappa_1 p' - \frac{1}{q} i_p = 0;$$

at $x = l$

$$E = 0; \qquad \varkappa_2 p' - \frac{1}{q} i_p = 0 \tag{9}$$

we determine the constants of integration

$$A_1 = B_1; \qquad A_2 = B_2;$$

$$A_3 = \frac{1}{M} \begin{vmatrix} G - \varkappa_1 B_3 - \dfrac{B_4}{q} - \mu p_0 \widetilde{E} & \varkappa_1 - \dfrac{1}{\tau^* \gamma_2} \\ \mu p_0 \widetilde{E} + \left( \dfrac{B_4}{q} - \varkappa_2 B_3 \right) e^{-\lambda l} & \left[ \varkappa_2 + \dfrac{1}{\tau^* \gamma_2} \right] e^{\gamma_2 l} \end{vmatrix};$$

$$A_4 = \frac{1}{M} \begin{vmatrix} \varkappa_1 - \dfrac{1}{\tau^* \gamma_1} & G - \varkappa_1 B_3 - \dfrac{B_4}{q} - \mu p_0 \widetilde{E} \\ \left[ \varkappa_2 + \dfrac{1}{\tau^* \gamma_1} \right] e^{\gamma_1 l} & \mu p_0 \widetilde{E} + \left( \dfrac{B_4}{q} - \varkappa_2 B_3 \right) e^{-\lambda l} \end{vmatrix}; \tag{10}$$

$$M = \begin{vmatrix} \varkappa_1 - \dfrac{1}{\tau^* \gamma_1} & \varkappa_1 - \dfrac{1}{\tau^* \gamma_2} \\ \left[ \varkappa_2 + \dfrac{1}{\tau^* \gamma_1} \right] e^{\gamma_1 l} & \left[ \varkappa_2 + \dfrac{1}{\tau^* \gamma_2} \right] e^{\gamma_2 l} \end{vmatrix}.$$

The quasiconstant $\widetilde{E}$ varies through the thickness of the semiconductor from the value $B_1$ as $x \to 0$ to the value $B_2$ as $x \to l$. Since $l \gg L''_d, L'_d$, then

$$\mathcal{E} = \frac{kT}{q} \frac{b-1}{b+1} \ln \frac{1 + \dfrac{b+1}{p_0 + b n_0} p'_0}{1 + \dfrac{b+1}{p_0 + b n_0} p'_l}. \tag{11}$$

In determining $p'_0$ and $p'_l$ from equations (7), one must bear in mind that the constants $\gamma_1, \gamma_2, B_i, A_i$ in turn depend on $p'_0, p'_l, i_{p0}$, and $i_{pl}$. Therefore equations (7) give only an implicit functional dependence of $p'_0$ and $p'_l$ on the intensity and on the parameters of the semiconductor, from which these quantities must still be determined in explicit form. However, it follows directly from the general formula (11) that, in addition to the normal Dember effect ($\mathcal{E} > 0$) for $p'_0 > p'_l$, an anomalous Dember effect ($\mathcal{E} < 0$) may also occur if $p'_0 < p'_l$. The sign of the Dember photo-emf is determined by the direction of the ambipolar diffusion current. In the case where this direction is created by the gradient of light intensity in the semiconductor, the photodiffusion current is directed from the illuminated face to the back face, and the normal Dember effect occurs. If, however, the light absorption coefficient is small (photogeneration of pairs occurs almost uniformly throughout the thickness of the sample) and, at the same time, the rate of surface recombination at the illuminated face is very large, then the concentration gradient of nonequilibrium charge carriers and the photodiffusion current will be directed from the shadow face to the illuminated face. In this case the Dember effect will be anomalous.

The formulas obtained above contain the stationary ($s = 0$), frequency-phase ($s = j\omega$), and transient (finding the originals by Laplace–

···of the characteristic of the Dember emf in semiconductors that are thick (in comparison with the Debye length), which makes it possible to carry out and interpret the experiment not only under steady, but also under sinusoidal and pulsed illumination of semiconductors.

3. Let us consider two particular cases corresponding to small generation levels ($p', p_0 \ll n_0$). For $s = 0,\ l \gg L,\ I = 0,\ G_0 \neq 0,\ x_1 \approx 0$, we obtain $p'_l = 0$ and

$$\mathcal{E} = \frac{kT}{q}\frac{b-1}{bn_0}\frac{L}{D}G_0. \tag{12}$$

i.e., the normal Dember effect. For $s = 0,\ G = 0,\ \lambda l \ll 1,\ \lambda_1 I_0 \neq 0,\ x_1 \to \infty;\ x_2 \approx 0$, we obtain $p'_0 = 0$ and

$$\mathcal{E} = -\frac{kT}{q}\frac{b-1}{bn_0}\lambda_1 I\tau\left(1 - \operatorname{sech}\frac{l}{L}\right), \tag{13}$$

i.e., the anomalous Dember effect.

The corresponding transient characteristics ($I,\ G = 0$ for $t < 0$; $I, G = I_0, G_0$ for $t > 0$) have the form [6,7]

$$\mathcal{E}(t) = \frac{2}{\sqrt{\pi}}\frac{kT}{q}\frac{b-1}{bn_0}\frac{L}{D}G_0\int_0^{\sqrt{t/\tau}} e^{-z^2}\,dz;$$

$$\mathcal{E}(t) = -\frac{kT}{q}\frac{b-1}{bn_0}\lambda_1 I_0\tau\left\{1 - \frac{4}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k(2k+1)}{(2k+1)^2 + \frac{4}{\pi^2}\frac{l^2}{L^2}}\left[1 - \exp\left(-\frac{1}{\tau} - \frac{\pi^2}{4}(2k+1)^2\frac{D}{l^2}\right)t\right]\right\}. \tag{14}$$

The transient characteristic of the anomalous Dember effect in this case can, with high accuracy, be approximated by the following formula [8,9]:

$$\mathcal{E}(t) = -\frac{kT}{q}\frac{b-1}{bn_0}\lambda_1 I_0 \tau\{1-h(t)\}, \tag{15}$$

where

$$h(t) = \begin{cases} 0, & \text{for } t \leq t_0, \\ A\left[\dfrac{1-e^{-a(t-t_0)}}{a} - \dfrac{1-e^{-b(t-t_0)}}{b}\right], & \text{for } t \geq t_0, \end{cases} \tag{16}$$

with

$$a = 2.467\frac{D}{l^2} + \frac{1}{\tau}; \qquad t_0 = 0.05\frac{l^2}{D};$$

$$b = 20.695\frac{D}{l^2} + \frac{1}{\tau}; \qquad A = 2.801\frac{D}{l^2}e^{-0.05l^2/D\tau}. \tag{17}$$

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*Note: Figure translations are in progress. See original paper for figures.*