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Abstract

Full Text

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ON THE MEASURE OF THE SET OF S-NUMBERS IN A p-ADIC FIELD

(Presented by Academician I. M. Vinogradov on 5 IV 1963)

Let $K(p)$ be the field of Hensel p-adic numbers, $\omega \in K(p)$, and let $|\omega|_p$ be the p-adic norm in $K(p)$. In K. Mahler' s classification (1) of transcendental numbers from $K(p)$, a number ω is called an S-number if there exists a real number $\upsilon > 0$ such that

$$|F(\omega)|_p > h^{-\upsilon}, \qquad h > h_0(n, \omega),$$

for every integral polynomial F of degree n and height h. Let $\upsilon_n(\omega)$ be the exact lower bound of those υ for which the indicated assertion is true.

D. Lock (2) proved that for almost all (in the sense of Turckstra measure (3)) numbers from $K(p)$ the relations

$$n + 1 \leq \upsilon_n(\omega) \leq 3n + 1 \qquad (n = 1, 2, ...)$$

hold. Later F. Kasch and B. Volkmann (4) obtained the inequalities

$$\upsilon_n(\omega) \leq 2n - \frac{1}{2} \qquad (n = 3, 4, ...),$$

as well as the equalities

$$\upsilon_1(\omega) = 2, \qquad \upsilon_2(\omega) = 3$$

for almost all numbers. At the same time, by analogy with Mahler' s known conjecture on real S-numbers, one may expect that

$$\upsilon_n(\omega) = n + 1 \qquad (n = 1, 2, ...)$$

for almost all numbers from $K(p)$.

We note that the arguments of Kasch and Volkmann (5) concerning the equality $\upsilon_3(\omega) = 4$ for almost all numbers are not correct.

Considerations analogous to those applied in the author' s papers ([6,7]) lead to the following results.

Theorem 1. *There exist numbers υ_n such that*

$$\upsilon_n(\omega) = \upsilon_n \qquad (n = 1, 2, ...)$$

for almost all p-adic numbers ω. The quantities υ_n satisfy the inequalities

$$n + 1 \leq \upsilon_n \leq \tfrac{5}{4}(n + \tfrac{1}{2}) \qquad (n = 3, 4, 5, 6, 7),$$

$$n + 1 \leq \upsilon_n \leq \tfrac{4}{3}n \qquad (n = 8, 9, ...).$$

Theorem 2. *The equality*

$$\upsilon_3(\omega) = 4$$

holds for almost all p-adic numbers ω.

For the proof of the last theorem, an estimate of the number of solutions of the Diophantine equation $x^3 = y^2 + A$, found in ([8]), is essential.

Detailed proofs are contained in the author' s dissertation submitted to Leningrad State University named after A. A. Zhdanov.

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Note: Figure translations are in progress. See original paper for figures.