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Fig. 1. Schematic view of a knot; $a = R/\rho$

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Abstract

Full Text

Physics

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Topological Structure of Space and the Mass Spectrum of Elementary Particles

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If one proceeds from the idea that all elementary particles are equally primary, then this permits, on the one hand, the rejection of the composite model, and, on the other hand, compels one to seek certain general properties and characteristics of matter and space which could, to some extent, allow a qualitative and quantitative explanation of the differences among particles. Such general characteristics could probably be the symmetry and structure properties of space—in other words, the metric of space.

It is known that the study of a spherically symmetric gravitational field leads to the result that, at a certain critical radius of a body, dependent on the mass of the body, the coordinate τ loses the property of being time-like. Contrary to the widespread opinion, it may be supposed that the loss by the coordinate τ of the property of being time-like means the impossibility of realizing a system with dimensions smaller than the critical ones.

Fig. 1. Schematic view of a knot
 $a = R/\rho$

Of course, critical surfaces may be of various types. In order for the transition of one body into another to be impeded, it is natural to require that the critical surfaces have different topology. It is thereby self-evident that the metric of space of a given topology must enter into the corresponding quantum equations, since all gravitational phenomena in elementary particles take place at distances smaller than the characteristic quantum distances.

Let us consider a class of different critical surfaces (we shall call them basic), having different topology and possessing a definite symmetry.

Below, in addition to spherical and axial symmetry, we shall also consider a group of “axial-screw” types of symmetry. Axial-screw symmetry is possessed by closed, simply connected surfaces twisted into knots, whose axial lines are

obtained by winding onto a directing torus (see Fig. 1). Below we shall simply call them knots.

The simplest basic surfaces are the sphere and the torus of circular cross section. As the next ones, it is natural to consider knots in which both the wound thread and the directing torus have circular cross sections.

One may try to find numerical relations between the characteristics of these simplest topological structures and the mass spectrum of elementary particles. For each topological structure possessing a definite symmetry, one can introduce its own quantitative characteristic—the packing coefficient. In this case, for a sphere and for a torus with zero inner radius, we define the packing coefficient as the ratio of the volume of the body to 1/8 of the volume of the parallelepiped described around it.

For all the following figures we shall define the packing coefficient as a quantity equal to the packing coefficient of a torus with zero inner radius, multiplied by the ratio of the volume of the knot with the given structure to the n -fold volume of a torus whose cross section is equal to the cross section of a strand of the knot, and whose axial line coincides with the axial line of the directing torus (n is the number of strands in the cross section of the knot).

According to this definition, the packing coefficient of any torus is equal to the packing coefficient of a torus with zero inner radius.

The lowest energy state under these assumptions should correspond to the surface with the greatest symmetry, i.e., a sphere. The packing coefficient for a sphere is

$$I_e = 4\pi/3. \quad (1)$$

The packing coefficient of a torus with circular cross section is

$$I_\pi = \pi^2/2. \quad (2)$$

The packing coefficients of more complicated figures, as calculation shows, are expressed by the formula

$$I_x = I_\pi \cdot \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \sqrt{\left(1 + \frac{\sin t}{a}\right)^2 + \frac{1}{a^2} \left(\frac{m}{n}\right)^2} dt, \quad (3)$$

where m and n are integers. The number n , defined above, is equal to the number of circuits around the axis of rotation z , while m is the number of circuits around the axial circle of the directing torus in the complete self-closing of the knot (see Fig. 1). The integrality of n and m is a necessary condition for the closing of the knot. The parameter a is equal to the ratio of the radius

of the axial circle of the directing torus to the radius of its cross section. We shall consider only such structures for which $m = n \pm 1$; this means that in one complete revolution around the axis z the cross section of the strand passes into the nearest neighboring one and at the same time makes, in addition, a full (if $m = n + 1$) or incomplete (if $m = n - 1$) revolution around the axis of the directing torus (see Fig. 1). We shall require that the parameter a take only integral values. Then the construction of a knot is, obviously, possible only for $a \geq 2$. (The scheme set forth uses knots with $a = 3$ and $a = 4$.)

Let us note that the packing coefficient of the torus I_π can be obtained from formula (3) for arbitrary a and for $n = 1$, $m = 0$, which corresponds to the fact that the packing coefficients of any tori of circular cross section are equal to one another. Thus formula (3) is, in a certain sense, a generalization of formula (2).

As was already noted by the author earlier ⁽¹⁾, for the mass of the electron the numerical relation

$$m_e = \frac{e}{\sqrt{G}} \exp \left\{ -\frac{\hbar c}{4e^2} \ln I_e \right\}, \quad (4)$$

is satisfied with high accuracy, where $I_e = 4\pi/3$ is the packing coefficient of the sphere, e is the charge of the electron, and G is the gravitational constant.

Recently Arnowitt, Deser, and Misner ⁽²⁾ showed within the framework of general relativity that a point charged particle must possess the mass

$$m = e/\sqrt{G}. \quad (5)$$

It seems possible to regard formula (5) as the classical limit ($\hbar \rightarrow 0$) of formula (4) and of all the relations given below.

The structure following the sphere (in terms of increasing complexity of the symmetry properties) is the torus. It turns out that for the mass of the π^\pm -meson the relation

$$m_{\pi^+} = m_e \exp \left\{ \frac{\hbar c}{4e^2} \ln \frac{I_\pi}{I_e} \right\}. \quad (6)$$

is satisfied with high accuracy.

More complicated topological structures—knots—are determined by two parameters: a and n .

It turns out that for the mass of the μ -meson the relation

$$m_\mu = m_{\pi^\pm} \exp \left\{ -\frac{\hbar c}{4e^2} \ln \frac{I_\mu}{I_\pi} \right\}, \quad (7)$$

holds, where I_μ is determined by formula (3) with $n = 2$, $m = 1$, $a = 4$.

For the mass of the K-mesons the relation

$$m_{K^\pm} = m_{\pi^\pm} \exp \left\{ \frac{\hbar c}{4e^2} \ln \frac{I_K}{I_\pi} \right\} \quad \text{for } a = 4, n = 11, m = 12. \quad (8)$$

holds.

Relations (4), (6), (7), (8) for the masses of e , π^\pm , μ^\pm , K^\pm , i.e., for all charged nonbaryons, are expressed by one and the same formula:

$$\frac{m_i}{m_{i+1}} = \exp \left\{ \pm \frac{\hbar c}{4e^2} \ln \frac{I_i}{I_{i+1}} \right\}. \quad (9)$$

In the chain of relations (9), for the electron there is no preceding particle and no preceding type of symmetry; therefore it may be assumed that the initial structure will be the structure with complete filling of space ($I_0 = 1$), and the corresponding initial mass is equal to the classical value ($m_0 = e/\sqrt{G}$), which corresponds to formula (4).

For the nucleon mass, a relation of type (9) does not exist, but the formula

$$m_n = m_{\pi^+} \exp \left\{ \frac{\hbar c}{4e^2} \left(\frac{I_N}{I_\pi} - 1 \right) \right\} \quad (10)$$

holds for $n = 3$, $m = 4$, $a = 4$.

The masses of all hyperons (Λ , Σ , Ξ) are obtained from a single formula:

$$m_Y = m_N \exp \left\{ \frac{\hbar c}{4e^2} \left(\left[\ln \left(\frac{I_Y}{I_\pi} \right) + 1 \right] \frac{I_\pi}{I_N} - 1 \right) \right\} \quad (11)$$

for the parameter values $a = 3$, $m_i = n_i + 1$, $n_\Lambda = 19$, $n_\Sigma = 14$, $n_\Xi = 10$. (In this case the mass values of the Λ^0 -, Σ^0 -, and Ξ^- -hyperons are obtained.)

It is curious that the choice of formulas (10), (9), and (11) is not independent in the full sense: if in formulas (9) and (11) $\ln(I_x/I_y)$ is replaced by the first term of the expansion $I_x/I_y - 1$, then formulas (9) and (11) take the form

$$m_x = m_y \exp \left\{ \pm \frac{\hbar c}{4e^2} \left(\frac{I_x}{I_y} - 1 \right) \right\}, \quad (12)$$

i.e., the same form as formula (10).

The numerical results are given in Table 1. The experimental data were taken from the review by Snow and Shapiro⁽³⁾. For the gravitational constant and the

fine-structure constant the values $G = 6.673 \cdot 10^{-8}$ CGSE, $\hbar c/e^2 = 137.0377 \pm 0.0016$ were used.

A feature of the formulas given is that they cover all isotopic multiplets (resonances were not considered), but the mass value of only one particle from the multiplet is obtained. Possibly this means that particles within a multiplet do not differ in topological structure, while the splitting occurs because of electromagnetic interaction. It is interesting that the values calculated from the formulas are somewhat greater than the experimental ones (the electron is an exception).

Thus, the constructed picture corresponds to the fact that the electron is assigned spherical symmetry; the π -meson, axial symmetry; the μ -meson, the lower axial-screw symmetry ($n = 2$); the nucleon, the next in complexity—axial-screw symmetry ($n = 3$); and strange particles (K-mesons and hyperons), very high types of axial-screw symmetry.

We have estimated the probability of a random coincidence of the values of mass ratios given by the formulas with experiment. In doing so, the accuracy

$$S_i = \left(\ln \frac{I_i}{I_\pi} + 1 \right) \frac{I_\pi}{I_N} - 1$$

Table 1

Particle	δ	Formula for δ	Structure	δ_{calc}	$\frac{\delta_{\text{calc}}}{\delta_{\text{exp}}} - 1$
e^\pm	$\frac{m_e \sqrt{G}}{e}$	$\exp \left\{ -\frac{\hbar c}{4e^2} \ln \frac{I_e}{I_\pi} \right\}$	Sphere	$(2.0552 \pm 0.0012) \cdot 10^{-21}$	$-(6.8 \pm 1.1) \cdot 10^{-3}$
π^\pm	$\frac{m_{\pi^\pm}}{m_e}$	$\exp \left\{ \frac{\hbar c}{4e^2} \ln \frac{I_{\pi^\pm}}{I_e} \right\}$	Torus	274.2	$+(3.6 \pm 0.4) \cdot 10^{-3}$
μ^\pm	$\frac{m_{\mu^\pm}}{m_{\pi^\pm}}$	$\exp \left\{ -\frac{\hbar c}{4e^2} \ln \frac{I_{\mu^\pm}}{I_{\pi^\pm}} \right\}$	$a = 4,$ $n = 2,$ $m = 1$	0.7603	$+(4.5 \pm 0.2) \cdot 10^{-3}$
K^\pm	$\frac{m_{K^\pm}}{m_{\pi^\pm}}$	$\exp \left\{ \frac{\hbar c}{4e^2} \ln \frac{I_{K^\pm}}{I_{\pi^\pm}} \right\}$	$a = 4,$ $n = 11,$ $m = 12$	3.5383	$+(1.1 \pm 0.8) \cdot 10^{-3}$
n	$\frac{m_n}{m_{\pi^\pm}}$	$\exp \left\{ \frac{\hbar c}{4e^2} \left(\frac{I_N}{I_{\pi^\pm}} \right) \right\}$	$a = 4,$ $n = 3,$ $m = 4$	6.7314	$+(1 \pm 4) \cdot 10^{-4}$

Particle	δ	Formula for δ	Structure	δ_{calc}	$\frac{\delta_{\text{calc}}}{\delta_{\text{exp}}} - 1$
Λ^0	$\frac{m_{\Lambda^0}}{m_n}$	$\exp\left\{\frac{\hbar c}{4e^2}S_{\Lambda}\right\}$	Knot, $a = 3,$ $n = 19,$ $m = 20$	1.1871	$+(2.3 \pm 1.4) \cdot 10^{-4}$
Σ^0	$\frac{m_{\Sigma^0}}{m_n}$	$\exp\left\{\frac{\hbar c}{4e^2}S_{\Sigma}\right\}$	Knot, $a = 3,$ $n = 14,$ $m = 15$	1.26924	$+(8.1 \pm 4.3) \cdot 10^{-4}$
Ξ^-	$\frac{m_{\Xi^-}}{m_n}$	$\exp\left\{\frac{\hbar c}{4e^2}S_{\Xi}\right\}$	Knot, $a = 3,$ $n = 10,$ $m = 11$	1.40636	$+(2.2 \pm 0.9) \cdot 10^{-3}$

formulas for the masses of the electron, the nucleon, and the cascade hyperon (Ξ^-) were not taken into account by us, since these particles are the “progenitors of the series,” and arbitrariness is built into the choice of the formula. We have two-parameter formulas. It turns out that, when the value of the parameter n is changed to the nearest value, the mass value changes by an amount $\sim 40 m_e$, while the deviation of the calculated value from the experimental one is $\sim 1 m_e$, i.e., the probability of a chance coincidence simultaneously at 5 points is $\sim \left(\frac{1}{40}\right)^5 \sim 10^{-8}$; a more accurate calculation, taking into account the deviations at each point and the intervals between mass values for the two nearest values of the parameters, gives for the probability of a chance coincidence of the calculated values with the experimental ones the quantity $5 \cdot 10^{-9}$.

If one adopts the point of view of the topological model set forth, then conservation of baryon charge means preservation of the structure with $n = 3$, $m = 4$ for the nucleon and the inevitable transition (for unclear reasons) into this structure of hyperon structures ($n \geq 10$). (The knots corresponding to antibaryons may have a mirror-reflected structure.) If this point of view is correct, then an unexpected qualitative consequence follows from it. The structures are in some way connected with the critical gravitational radius. If a gravitational field close to the critical one is imposed, then the structures may become unstable, i.e., in superstrong gravitational fields the law of conservation of baryon number may be violated. Such a violation, not observed under terrestrial conditions, may be associated with a large release of energy in superdense or superheavy systems, for example in supernova stars.

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