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Abstract

Full Text

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THE INFLUENCE OF CHANGES IN THE PARAMETERS OF LINEAR GYROSCOPIC SYSTEMS ON THE FREQUENCIES OF OSCILLATIONS AND THE DAMPING COEFFICIENTS

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The roots of the characteristic equation of a gyroscopic system satisfy equality (1)

$$T^*\mu^2 + (D^* + i\Gamma^*)\mu + V^* + iE^* = 0, \tag{1}$$

where $T^*, D^*, \Gamma^*, V^*, E^*$ are real numbers:

$$T^* = \sum\sum a_{ik}A_iA'_k, \qquad V^* = \sum\sum c_{ik}A_iA'_k, \qquad D^* = \sum\sum b_{ik}A_iA'_k,$$

$$i\Gamma^* = \sum\sum \gamma_{ik}A_iA'_k, \qquad iE^* = \sum\sum \varepsilon_{ik}A_sA'_k, \tag{2}$$

where

$$a_{ik} = a_{ki}, \qquad b_{ik} = b_{ki}, \qquad c_{ik} = c_{ki}, \qquad \gamma_{ik} = -\gamma_{ki}, \qquad \varepsilon_{ik} = -\varepsilon_{ki}.$$

The conjugate constants A_i, A'_i satisfy the system of equations

$$\sum \left(a_{ik}\mu^2 + (b_{ik} + \gamma_{ik})\mu + c_{ik} + \varepsilon_{ik}\right) A_k = 0$$

or

$$\sum \left[a_{ik}{\mu'}^2 + (b_{ik} + \gamma_{ik})\mu' + c_{ik} + \varepsilon_{ik}\right] A'_k = 0 \qquad (i = 1, 2, \dots, n). \tag{3}$$

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

From (3) it is clear that A_i depend on the roots of the characteristic equation, and from (1) we find that the roots of the characteristic equation depend on A_k and on the coefficients $a_{ik}, \dots, \varepsilon_{ik}$.

It is easy, however, to prove that the roots of the characteristic equation do not depend on A_k, A_k'; therefore, when determining the differential $d\mu$ from (1), one should take into account the dependence of $T^*, \dots, E^*$ and μ only on the coefficients $a_{ik}, \dots, \varepsilon_{ik}$; this is readily checked directly if one takes into account relations (3).

This observation is the basis of a method for investigating the influence of changes in the coefficients $a_{ik}, \dots, \varepsilon_{ik}$ on the frequencies of oscillations and on the damping coefficients.

Restricting ourselves to such gyroscopic systems in which the gyroscopic forces have a dominant significance ([1]), let us recall that the roots of equation (1)

$$\mu = \frac{-(D + i\Gamma) \pm (x + iy)}{2T},$$

$$\mu_1 = \frac{-(D - x) - i(\Gamma - y)}{2T}, \qquad \mu_2 = \frac{-(D + x) - i(\Gamma + y)}{2T}, \tag{4}$$

have negative coefficients of the imaginary unit if the system is statically unstable ($V^* < 0$). If the system is statically stable ($V^* > 0$),

then the root μ_1 may be positive, since

$$\begin{aligned} \mu_1 &\simeq -\frac{D^* - x}{2T} + \frac{4V^*T^* - D^{*2}}{4TT^*} i, \\ \mu_2 &\simeq \frac{D^* + x}{2T^*} - \left(\frac{\Gamma^*}{T^*} + \frac{4V^*T^* - D^{*2}}{4T^*T^*} \right) i. \end{aligned} \tag{5}$$

As for Γ^* and E^*, both functions may be regarded as positive or negative, but necessarily of the same sign ([1]). With simultaneous changes in the signs of Γ^* and E^*, the roots of equation (1) do not change.

Fig. 1

Fig. 2

A change in the potential energy of the system entails a change in the roots of the characteristic equation, which can be determined from equality (1).

If the gyroscopic system is conservative ($D^* \equiv 0$ and $\Gamma^* \equiv 0$), then

$$\frac{\partial\mu_1}{\partial c} = \frac{\partial V^*/\partial c}{+\sqrt{\Gamma^{*2} + 4V^*T^*}}\,i, \qquad \frac{\partial\mu_2}{\partial c} = \frac{\partial V^*/\partial c}{-\sqrt{\Gamma^{*2} + 4V^*T^*}}\,i,$$

where $c = c_{ik}$.

From the last equalities it follows: **with an increase in the stiffness of an ideal gyroscopic system, its oscillation frequencies increase.**

The proof is readily seen from Fig. 1a.

If the masses (the coefficients a_{ik}) in an ideal gyroscopic system are increased, then

$$\frac{\partial\mu_1}{\partial a} = +\frac{\dfrac{\partial T^*}{\partial a}\,\mu^2}{\sqrt{\Gamma^{*2} + 4V^*T^*}}\,i, \qquad \frac{\partial\mu_2}{\partial a} = -\frac{\dfrac{\partial T^*}{\partial a}\,\mu^2}{\sqrt{\Gamma^{*2} + 4V^*T^*}}\,i.$$

Assuming that the system is statically stable, and noting that $\mu^2 = -\beta^2 < 0$, we arrive at the conclusion: **with an increase in the masses of the system, the oscillation frequencies decrease, if the system is statically stable.**

For a statically unstable system ($V^* < 0$), the frequencies of the slow oscillations increase, while the frequencies of the fast oscillations decrease (Fig. 1b) with increasing masses of the system.

The influence of a change in stiffness in a nonconservative gyroscopic system ($D^* \neq 0,\ E^* \neq 0$) can be investigated if $\partial\mu/\partial c$ ($c = c_{ik}$) is determined from equality (1):

$$\begin{aligned} \frac{\partial\mu_1}{\partial c} &= -\frac{\partial V^*/\partial c}{2T^*\mu + D^* + i\Gamma^*} = -\frac{\partial V^*/\partial c}{x^2 + y^2}(x - iy), \\ \frac{\partial\mu_2}{\partial c} &= -\frac{\partial V^*/\partial c}{-(x + iy)} = \frac{\partial V^*/\partial c}{x^2 + y^2}(x - iy). \end{aligned} \tag{6}$$

Figure 2 presents the vectors $\partial\mu_1/\partial c$ and $\partial\mu_2/\partial c$ for a statically stable system ($V^* > 0$). Hence one may draw the following conclusion: **with an increase in stiffness, the damping coefficients and the frequencies of the slow oscillations increase; the frequencies of the fast oscillations increase, while the damping coefficients decrease.**

Replacing in equalities (6) the derivative $\dfrac{\partial V^*}{\partial c}$ by $\dfrac{\partial E^*}{\partial \varepsilon} i$ ($\varepsilon = \varepsilon_{ik}$), we obtain

Fig. 3

Fig. 3

Figure 3: Fig. 3

$$
\begin{aligned}
\frac{\partial\mu_1}{\partial\varepsilon} &= -\frac{\frac{\partial E^*}{\partial\varepsilon} i}{x^2+y^2}(x-iy), \\
\frac{\partial\mu_2}{\partial\varepsilon} &= -\frac{\frac{\partial E^*}{\partial\varepsilon} i}{x^2+y^2}(x-iy).
\end{aligned} \tag{7}
$$

Using Fig. 3, which shows the vectors $\partial\mu_1/\partial\varepsilon$ and $\partial\mu_2/\partial\varepsilon$, we find: **with an increase in the intrinsic nonconservative forces, the oscillation frequencies decrease; the damping coefficients of the slow oscillations increase, while those of the fast oscillations decrease.**

Obviously, by the indicated method one can also prove a number of other, more complicated theorems, if various assumptions are made concerning changes in the parameters of the system.

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1. I. I. Metelitsyn, DAN, **86**, No. 1 (1952).

Note: Figure translations are in progress. See original paper for figures.