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**Abstract**

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### PHYSICS

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# SHEAR WAVES OF FINITE AMPLITUDE IN POLY- AND SINGLE CRYSTALS OF METALS

*(Presented by Academician M. A. Leontovich, 13 XII 1962)*

In previous works ([1,2]) a nonlinear distortion of longitudinal ultrasonic waves was established experimentally; the principal cause of this distortion is the deviation from Hooke' s law, connected ultimately with the nonlinear dependence of the intermolecular interaction forces on the displacement of atoms (or ions) from the equilibrium position ([3]). In ([1]) the distortion of shear waves was not observed because of its relatively small magnitude. Subsequent, more careful observations, the results of which are presented in this article, show that nonlinear distortions of shear waves do exist.

According to the nonlinear theory of elasticity, in isotropic solids in the second approximation no distortion of plane shear waves should be observed ([4]). This is connected with the fact that in the nonlinear Hooke' s law for shear deformation there is no quadratic term because of the equivalence of shear in the forward and reverse directions*. As for crystals, for example, in crystals with the highest symmetry (the cubic system, class *m3m*), there are only a few directions in which the effective shear nonlinearity for a plane wave, determining the occurrence of a harmonic, is equal to zero, i.e., there will be no second harmonic in these directions, just as in an isotropic body. For crystals of class *m3m* such directions will be, for example, the axes [100], [010], [001] and the face diagonals [110], [101] and [011]. The sign, as well as the magnitude, of the nonlinear shear coefficient in other directions in an *m3m* crystal depends on the direction of sound propagation. In the case where, in addition to a shear wave, a longitudinal wave is emitted into an isotropic body, it is not difficult, using the equations of the theory of elasticity in the second approximation, to show that there is no continuously increasing solution in space for the second shear harmonic: the amplitude of the harmonic, because of the absence of synchronism (the velocity of the driving force differs from the velocity of the emerging transverse wave), varies periodically in space, the period of variation of the harmonic amplitude being of the order of the length of the longitudinal wave.

Oscillogram

Figure 1: Oscillogram

In this work we report experimental results on the observation of a shear harmonic generated by the shear wave itself in polycrystalline metals (magnesium-aluminum alloy MA-8, aluminum, and duralumin) and in aluminum single crystals; qualitative observations were also carried out in single crystals of zinc and cadmium. The distortion of transverse ultrasonic waves (5 MHz) was determined from the appearance in the signal that had passed through the metallic specimen of the second harmonic. The latter, as in the case of longitudinal waves ([1,2]), was isolated by a quartz receiver with the resonance frequency of the second harmonic (10 MHz), by an electrical filter—

* It should be said that the quantum laws of conservation of energy and momentum of phonons allow the possibility of a "merging" of two transverse phonons with identical directions of momentum, as a result of which a transverse phonon of the total energy is obtained. However, because of the form of the expression for the elastic energy, the probability of such an interaction is equal to zero.

by a 5 MHz probe, a resonant amplifier with a gain of $10^3$. Transverse waves were emitted and received by BT-cut quartz plates glued to the specimen with picein; identical polarization of the emitter and receiver oscillations was carefully selected. Most observations were carried out with a pulse amplitude at the emitter of 1000 V, which corresponds to an amplitude of shear stresses of several kg/cm$^2$. Under the conditions of our experiment, the BT-cut quartz emitted, in addition to the shear wave, also a parasitic longitudinal wave, whose amplitude was approximately 5 times

**Fig. 1.** Oscillogram of second-harmonic pulses in an MA-8 specimen 10 cm long. $M$—pickup; $L_1, L_2, ...$—pulses of the longitudinal second harmonic produced by the parasitic longitudinal wave; $S_1, S_2, ...$—pulses of the second harmonic of shear.

smaller than the amplitude of the shear wave. As in the case of longitudinal waves, in some materials (for example in MA-8) oscillograms characteristic of accumulating effects were observed (Fig. 1): the first pulse $S_1$, which had traversed one specimen length, was smaller than the second $S_2$, which had traversed triple the length; the subsequent pulses $S_3, S_4, ...$ decreased. This is explained by the fact that the amplitude of the second harmonic grows up to the stabilization distance and then decreases. The propagation velocity of the observed second-harmonic signals was equal to the propagation velocity of the shear wave. These circumstances may be regarded as one confirmation that the observed shear harmonic is not the result of interaction of the emitted shear wave with the parasitic longitudinal wave.

Figure 2

Figure 2: Figure 2

The mean effective nonlinear coefficient for longitudinal waves has been determined for some metals and alkali-halide crystals in ([2],[3] ); its value for different materials varies within the limits $(3 \div 10)(K + 4/3\mu)$, where $K$ is the bulk modulus and $\mu$ is the shear modulus. These results agree, in order of magnitude, with data from static measurements, and also with the results of calculation on the basis of the simplest model of a solid. The latter circumstance permits one to consider that the principal cause of distortion of longitudinal waves in solids is the nonlinear dependence of intermolecular forces on the relative displacement of atoms (or ions). In the case of shear waves, according to our measurements, the effective nonlinear coefficient in the investigated metals is two to three orders of magnitude smaller*, which may be regarded as an indication that the cause of shear nonlinearity differs from the cause of nonlinearity for lon–

* A more accurate determination of the shear nonlinear coefficient is associated with the difficulty of measuring the absolute value of the shear stresses in the wave.

longitudinal waves. A very convincing argument in favor of this is also the sensitivity, observed by us, of the nonlinear shear coefficient of crystalline aluminum to small local static loads and slight heating, which will be discussed below.

In single crystals of aluminum, belonging to the class $m3m$, as was already noted above, for an arbitrary direction of propagation of the shear wave relative to the crystallographic axes the nonlinear coefficient is, generally speaking, different from zero. We investigated specimens cut from single crystals in such a way that: 1) the axis of the specimen was oriented along the cubic axis (in this specimen, as already noted above, the second shear harmonic should not have been observed); the size of the specimen was $6 \times 1.5 \times 1.5$ cm; 2) several specimens in which the axis did not coincide with the cubic axis, with dimensions $13 \times 1.5 \times 1.5$ cm*. The second shear harmonic was observed both in the specimen oriented along the cubic axis and in the other specimens. Thus here, as in the case of isotropic bodies, the observed appearance of a harmonic during propagation along the cubic axis is not consistent with the theory of elasticity of crystals.

**Fig. 2.** Dependence of the relative amplitude of the second shear harmonic in an aluminum single crystal on the load applied to a pad of 12 mm$^2$; $x$ is the distance of the center of the pad from the sound source. The specimen is oriented along the [100] axis, and the force acts along the [001] axis.

In single crystals of aluminum, cadmium, and zinc, the amplitude of the second shear harmonic proved sensitive to small static loads. The loads were applied as follows: 1) tension along the direction of ultrasound propagation; 2) unilateral compression under a load of up to 8 kgf of the side faces of the specimen at different points; in this case it turned out that the change in the harmonic, at least for an insignificant change in the area of application of the force, does not depend on the pressure (the load was applied to pads of 1 and 12 $mm^2$), but depends on the force; in addition, a certain dependence on the clamping conditions of the specimen during fastening was observed (the free specimen was most sensitive to the load). The dependence of the harmonic on force, and not on pressure, is evidently connected with the fact that increasing the area of application of the force at constant load also leads to an increase in the volume of the deformed region, while the observed effect is integral. The results of measuring the dependence of the relative amplitude of the shear harmonic (the amplitude of the harmonic without load is taken as unity) on the applied load are shown in Fig. 2.

At most points of application the load increases the harmonic; at some points, sometimes only at small loads, the opposite dependence is observed. The change in the amplitude of the shear harmonic under small loads is reversible: when the load is removed, the harmonic amplitude in aluminum returns to its initial value fairly rapidly. In the case of deformation by weak bending in cadmium, the return to the initial value after removal of the load occurs substantially more slowly ($\sim$ 10–15 s). After repeated application of loads, in some cases a peculiar “fatigue” was observed, expressed in a decrease of the effect.

* The authors take this opportunity to express their gratitude to I. V. Telegina for determining, by the X-ray method, the orientation of the aluminum single-crystal specimens.

It should be said that small loads slightly increased the damping of the first shear harmonic in an aluminum single crystal (this was observed at first-harmonic frequencies of 5 and 10 MHz), whereas the second shear harmonic increased. Since the change in the speed of sound and in the density of the metal under such small local loads is insignificant, it may be said that the change in the amplitude of the harmonic is associated mainly with the change, under the action of a small load, of the effective shear nonlinear coefficient.

Qualitative observations show that short-term (on the order of several seconds) heating of a specimen in a gas flame, or even heating with a match, substantially (by a factor of 2–3) changes (as a rule increases) the amplitude of the shear harmonic in single crystals of aluminum, zinc, and cadmium, whereas the first harmonic is not changed by this heating. This also gives grounds to suppose that short-term heating substantially changes the effective shear nonlinearity of single crystals.

The influence of small loads and heating was observed only in single crystals and only for nonlinear distortion of shear waves. In polycrystalline aluminum even severe cold working did not change the amplitude of the second shear harmonic to any substantial extent. Even larger loads did not exert any significant influence on the effective nonlinearity of longitudinal waves in single crystals.

Thus, in summary, it may be said that, in contrast to the nonlinearity of compression–rarefaction, where external pressures (up to the very highest, judging from Bridgman' s results for hydrostatic compression) or heating (to temperatures substantially lower than the melting point) have practically no effect on the magnitude of the nonlinear coefficient, the nonlinear elasticity for shear in single crystals proves to be extremely sensitive to weak external actions: small loads and slight heating. All this indicates that shear nonlinearity is determined to a considerable extent by metastable elastic states of single crystals. The experimental observation of a shear harmonic in those cases in which, according to the theory of elasticity, it should not have been observed (in polycrystalline isotropic materials and in a cubic crystal along an axis) indicates that shear nonlinearity is determined by the deviation of real solids from the model used in the theory of elasticity. At present it is difficult to say what type of deviations play the principal role in the distortion of shear waves. The slip stress in pure aluminum single crystals is of the order of several kg/cm$^2$, i.e., of the order of the static stresses applied in our experiment, and also of the order of the alternating stresses in the wave. Consequently, it may be assumed that shear nonlinearity is at least partly determined by the presence of dislocations. The question of the influence of residual internal stresses, which may possibly also lead to shear anisotropy and to sensitivity of the nonlinear coefficients to weak external actions (loads and heating), is not yet entirely clear. Naturally, these factors must exert some influence not only on shear nonlinearity, but also on the nonlinearity for longitudinal waves. However, the distortion of longitudinal waves associated with dislocations and residual stresses is almost not manifested against the background of the substantially larger distortion caused by the nonlinearity of intermolecular interaction.

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*Note: Figure translations are in progress. See original paper for figures.*