MECHANICS

D. MANZHERON, E. KROITOR

1963

SovietRxiv

Abstract

Full Text

MECHANICS

D. MANZHERON, E. KROITOR

ON THE GENERAL THEORY OF REDUCED ACCELERATIONS IN TANGENTIAL COORDINATES

(Presented by Academician I. I. Artobolevskii, 13 VII 1962)

I. I. Artobolevskii investigated, in a number of recently published articles ([1,2]), a class of linearly enveloping coupler curves important for applications, obtaining their Cartesian equations in parametric form. In ([3]), one of the authors of the present note pointed out the advantages of the systematic use of the methods of tangential geometry in investigations concerning: 1) envelopes of certain lines connected with the links of mechanisms in plane-parallel motion; 2) envelopes of certain planes connected with the links of mechanisms in general spatial motion; 3) dual problems, i.e., ruled surfaces formed by certain lines connected with the links of mechanisms in general spatial motion.

1. Let

$$\mathbf{r}_M = \mathbf{r}_{M_0} + \mu_1\mathbf{u}_1 + \mu_2\mathbf{u}_2, \qquad (\mathbf{u}_1 \cdot \mathbf{u}_2) = 0, \qquad \mathbf{u}_1^2 = \mathbf{u}_2^2 = 1, \tag{1}$$

be the point-vector equation of a certain moving plane (P), rigidly connected with some link of a mechanism in general spatial motion, and

$$\mathbf{r}_{M*} = \mathbf{r}_{M_0} + \mu_1\mathbf{u}_1 + \mu_2\mathbf{u}_2 + \lambda\frac{d^{m+1}}{dt^{m+1}}(\mathbf{r}_{M_0} + \mu_1\mathbf{u}_1 + \mu_2\mathbf{u}_2) \equiv \mathbf{r}_M + \lambda\mathbf{a}_M^{(m)} \tag{2}$$

be the point-vector equation of the plane (P_m)—the locus of the endpoints of accelerations of arbitrary order $\mathbf{a}_M^{(m)}$ of points $M \in (P)$, multiplied by a certain factor λ, where

$$\mathbf{a}_M^{(m)} = \mathbf{a}_{M_0}^{(m)} + (-A_m + \mathbf{B}_m\times)\mathbf{u} + \sum_{j=0}^{m-1}\vec{\omega}^{(j)}(C_{mj}\cdot\mathbf{u}), \tag{3}$$

where

$$A_{m+1} = \frac{dA_m}{dt} + \vec{\omega} \cdot \mathbf{B}_m,$$

$$A_1(\mathbf{u}) = (\vec{\omega} \cdot \mathbf{u})(\vec{\omega} \cdot \mathbf{u}) \quad \text{for} \quad A_1 = (\vec{\omega} \cdot \vec{\omega}), \tag{4}$$

$$A_2(\mathbf{u}) = 3(\vec{\omega}^{(2)} \cdot \mathbf{u})(\vec{\omega} \cdot \mathbf{u}) \quad \text{for} \quad A_2 = 3(\vec{\omega}^{(2)} \cdot \vec{\omega}) \text{ etc.},$$

$$\mathbf{B}_{m+1} = \frac{d\mathbf{B}_m}{dt} - A_m\vec{\omega},$$

$$B_1(\mathbf{u}) = (\mathbf{B}_1 \cdot \mathbf{u}) = (\vec{\omega}^{(2)} \cdot \mathbf{u}) \quad \text{for} \quad \mathbf{B}_1 = \vec{\omega}^{(2)}, \tag{5}$$

$$B_2(\mathbf{u}) = (\mathbf{B}_2 \cdot \mathbf{u}) = (\vec{\omega}^{(3)} \cdot \mathbf{u}) - (\vec{\omega} \cdot \mathbf{u})(\vec{\omega} \cdot \vec{\omega}) \quad \text{for} \quad \mathbf{B}_2 = \vec{\omega}^{(3)} - (\vec{\omega} \cdot \vec{\omega})\vec{\omega} \text{ etc.};$$

$$C_{mj} = C_{m-1,j-1} + DC_{m-1,j} \quad (m = 1, 2, \ldots;\ j = 0, 1, \ldots, m-1), \tag{6}$$

$$C_{10} = \vec{\omega}, \qquad C_{m-1,-1} = \mathbf{B}_{m-1}, \qquad D \equiv \frac{d}{dt} - \vec{\omega} \times .$$

μ_1, μ_2 are parameters, $\mathbf{u} = \mu_1\mathbf{u}_1 + \mu_2\mathbf{u}_2$, and $\vec{\omega} = \vec{\omega}(t)$ is the pseudovector of the instantaneous angular velocity of the plane (P).

The problem of determining the values of the parameter λ for which some trigonometric function of the dihedral angle formed by the planes (P) and (P_m) attains an extremum leads to the values

$$\lambda'_m = \frac{1}{2}\,\frac{A_m + A_m(\mathbf{u}_3) + \sqrt{(A_m - A_m(\mathbf{u}_3))^2 - 4B_m^2(\mathbf{u}_3)}}{A_m \cdot A_m(\mathbf{u}_3) + B_m^2(\mathbf{u}_3)}, \tag{7}$$

$$\lambda''_m = \frac{1}{2}\,\frac{A_m + A_m(\mathbf{u}_3) - \sqrt{(A_m - A_m(\mathbf{u}_3))^2 - 4B_m^2(\mathbf{u}_3)}}{A_m \cdot A_m(\mathbf{u}_3) + B_m^2(\mathbf{u}_3)}, \tag{8}$$

$$\mathbf{u}_3 = \mathbf{u}_1 \times \mathbf{u}_2,$$

and the following theorems hold.

Theorem 1. The planes (P) and (P'_m), or (P) and (P''_m),

$$(P), (P'_m) \qquad \mathbf{r}'_{M^*} = \mathbf{r}_{M_0} + \mu_1 \mathbf{u}_1 + \mu_2 \mathbf{u}_2 + \lambda'_m \mathbf{a}_M^{(m)}, \tag{9}$$

$$(P), (P''_m) \qquad \mathbf{r}''_{M^*} = \mathbf{r}_{M_0} + \mu_1 \mathbf{u}_1 + \mu_2 \mathbf{u}_2 + \lambda''_m \mathbf{a}_M^{(m)}, \tag{10}$$

with the set of vectors of reduced accelerations of any order, defined by the equalities

$$\mathbf{a}_{Mr}^{(m)'} = \lambda'_m \mathbf{a}_M^{(m)}; \qquad \mathbf{a}_{Mr}^{(m)''} = \lambda''_m \mathbf{a}_M^{(m)}, \tag{11}$$

and having their origins on the plane (P) and their endpoints on the plane (P'_m), or on (P''_m), form generalized spatial Kotelnikov crosses ([4]) corresponding to the chosen order m.

Theorem 2. The triples of planes (P), (P'_m), (P''_m) form, by their intersections, right-angled trihedra.

2. Let u_1, u_2, u_3 be the tangential coordinates of a certain plane whose Cartesian equation is

$$u_1 x_1 + u_2 x_2 + u_3 x_3 + 1 = 0. \tag{12}$$

The tangential coordinates of the principal planes (P), (P'_m), or (P), (P''_m) of the generalized Kotelnikov crosses are expressed by the matrix relations

$$\left\{ \begin{matrix} u'_1 \\ u'_2 \\ u'_3 \end{matrix} \right\} = - \left\{ \begin{matrix} g'_1 \\ g'_2 \\ g'_3 \end{matrix} \right\} F', \qquad F' = (\mathbf{r}_{M_0} \cdot \mathbf{g}') - \lambda'_m (\mathbf{r}_{M_0}^{(m)} \cdot \mathbf{g}'), \qquad \mathbf{g}' \equiv (g'_1, g'_2, g'_3), \tag{13}$$

$$g'_1 = v'_{12} v'_{23} - v'_{13} v'_{22}, \qquad g'_2 = v'_{13} v'_{21} - v'_{11} v'_{23}, \qquad g'_3 = v'_{11} v'_{22} - v'_{12} v'_{21}, \tag{14}$$

$$v'_{lk} = (\mathbf{u}_l \cdot \mathbf{i}_k)(1 - \lambda'_m A_m) + \lambda'_m (B_m \mathbf{u}_l \cdot \mathbf{i}_k) + \lambda'_m \sum_{j=0}^{m-1} (\vec{\omega}^{(j)} \cdot \mathbf{i}_k)(C_{mj} \cdot \mathbf{u}_l)$$

$$(l = 1, 2; \; k = 1, 2, 3);$$

$$\left\{\begin{array}{c} u_1'' \\ u_2'' \\ u_3'' \end{array}\right\} = -\left\{\begin{array}{c} g_1'' \\ g_2'' \\ g_3'' \end{array}\right\} F'', \qquad F'' = (\mathbf{r}_{M_0} \cdot \mathbf{g}'') - \lambda_m''(\mathbf{r}_{M_0}^{(m)} \cdot \mathbf{g}''); \qquad \mathbf{g}'' \equiv (g_1'', g_2'', g_3''); \tag{15}$$

$$g_1'' = v_{12}''v_{23}'' - v_{13}''v_{22}'', \qquad g_2'' = v_{13}''v_{21}'' - v_{11}''v_{23}'', \qquad g_3'' = v_{11}''v_{22}'' - v_{12}''v_{21}'',$$

$$v_{lk}'' = (\mathbf{u}_l \cdot \mathbf{i}_k)(1 - \lambda_m'' A_m) + \lambda_m''(B_m \mathbf{u}_l \cdot \mathbf{i}_k) + \lambda_m'' \sum_{j=0}^{m-1} (\vec{\omega}^{(j)} \cdot \mathbf{i}_k)(C_{mj} \cdot \mathbf{u}_l) \tag{16}$$

$$(l = 1, 2;\ k = 1, 2, 3),$$

where $\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3$ are the unit vectors of the coordinate axes of the "fixed" trihedron.

3. In an analogous manner one can obtain, starting from the point-vector equations of a certain line (D) in complex spatial motion,

$$\mathbf{r}_M = \mathbf{r}_{M_0} + \mu(a\mathbf{u}_1 + b\mathbf{u}_2 + c\mathbf{u}_3), \qquad (\mathbf{u}_i \cdot \mathbf{u}_j) = \delta_{ij}, \qquad \mathbf{u}_1^2 = \mathbf{u}_2^2 = \mathbf{u}_3^2 = 1,$$

$$a^2 + b^2 + c^2 = 1 \tag{17}$$

$$(i, j = 1, 2, 3;\ \delta_{ij} \text{ is the Kronecker symbol})$$

and from the associated line (D_m)

$$\mathbf{r}_{M^*} = \mathbf{r}_M + \lambda_m \mathbf{a}_M^{(m)}, \tag{18}$$

where

$$\mathbf{a}_M^{(m)} = \mathbf{a}_{M_0}^{(m)} + \mu(-\mathbf{A}_m + \mathbf{B}_m \times)(a\mathbf{u}_1 + b\mathbf{u}_2 + c\mathbf{u}_3) +$$

$$+\mu \sum_{j=0}^{m-1} \omega^{(j)}(\mathbf{C}_{mj} \cdot (a\mathbf{u}_1 + b\mathbf{u}_2 + c\mathbf{u}_3)). \tag{19}$$

and $\vec{\omega} = \vec{\omega}(t)$ is the pseudovector of the instantaneous angular velocity of the line (D), and the symbols entering into (19) are defined by (4)–(6), the theorem dual to Theorem 1, which leads to generalized dual Kotelnikov crosses. In this case the principal lines of one of such crosses, corresponding to the chosen order m of the reduced acceleration $\mathbf{a}_{M_r}^{(m)}$,

$$\mathbf{a}_{M_r}^{(m)} = \frac{\mathbf{a}_M^{(m)}}{A_m - A_m(a\mathbf{u}_1 + b\mathbf{u}_2 + c\mathbf{u}_3)}, \tag{20}$$

$$(D),\ (D_{M^*}) \qquad \mathbf{r}_{M^*} = \mathbf{r}_M + \frac{\mathbf{a}_M^{(m)}}{A_m - A_m(a\mathbf{u}_1 + b\mathbf{u}_2 + c\mathbf{u}_3)} \tag{21}$$

($A_m(a\mathbf{u}_1 + b\mathbf{u}_2 + c\mathbf{u}_3)$ is determined by "polarization with respect to $\mathbf{u} = a\mathbf{u}_1 + b\mathbf{u}_2 + c\mathbf{u}_3$" according to formulas (4)), can be expressed in tangential coordinates, starting from the tangential equations of two of their points

$$a_1u_1 + a_2u_2 + a_3u_3 + 1 = 0, \qquad b_1u_1 + b_2u_2 + b_3u_3 + 1 = 0. \tag{22}$$

4. The results obtained, which can also be generalized to "Kotelnikov crosses" consisting of certain associated pairs of linear subspaces V_{n-k} of an n-dimensional Euclidean space E_n, may be used (as is done directly in paper [5] and in some other studies [6]) to determine parametric equations in tangential coordinates of linearly enveloped curves or else ruled surfaces, and then also to determine velocities and accelerations of various orders of planar and spatial mechanisms; thus one can supplement the previously developed grapho-analytical methods of reduced accelerations of any order, which have found application in a number of recently carried out works by Soviet and Romanian authors [7–12].

Iași Polytechnic Institute
Iași, Romania

Received
26 IV 1962

CITED LITERATURE

[1] I. I. Artobolevskii, DAN, 139, No. 4, 838 (1961).
[2] I. I. Artobolevskii, DAN, 139, No. 5, 1077 (1961).
[3] D. Mangeron, Izv. AN SSSR, Mechanics and Mechanical Engineering, No. 3 (1962).
[4] A. P. Kotelnikov, Tr. Moscow Mechanical-Engineering Institute named after N. Bauman, issue 29–30, 3 (1937).
[5] D. Mangeron, E. Croitoru, Atti Accad. Naz. dei Lincei, Rend. Cl. sci. fis., mat. e nat., s. VIII, 32, 5 (1962).
[6] D. Mangeron, D. S. Tavkhelidze, Bul. Inst. politehn. Iași, S. N., 7 (11), 3

(1962).
[7] I. I. Artobolevskii, Tr. II All-Union Conference on the Principal Problems of the Theory of Machines and Mechanisms, Analysis and Synthesis of Mechanisms, 1960, p. 8; III Gesamtpolnische Konf. über Theorie der Getrieben und Maschinen, Rogów —Warschau, 1 Juni, 1961, Diskussionen.
[8] N. I. Levitskii, Results of the II All-Union Conference on the Principal Problems of the Theory of Machines and Mechanisms, Publishing House of the Academy of Sciences of the USSR, 1958, p. 24.
[9] A. I. Yasyulenis, Lietuvos Zemes Ukio Akademijos Moksliniai Darbai, 5 (1959).
[10] Ya. S. Zilberman, Tr. Rostov-on-Don Institute of Railway Transport Engineers, 1961.
[11] D. Mangeron, C. Drăgan, Rev. de Mecanique appl., 7, 2 (1962).
[12] D. Mangeron, R. Bogdan, Stud. și cercetări Mec. apl., 9, 4 (1958).

Note: Figure translations are in progress. See original paper for figures.