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Abstract

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P. V. KHARLAMOV

ONE CASE OF INTEGRABILITY OF THE EQUATIONS OF MOTION OF A HEAVY RIGID BODY HAVING CAVITIES FILLED WITH LIQUID

(Presented by Academician P. Ya. Kochina on 29 XII 1962)

Consider a rigid body having a fixed point and cavities (among them also multiply connected ones) filled with an ideal liquid. In the general case the center of gravity of this system does not coincide with the point of support.

As is known ([1]), the velocity potential of the liquid filling can be represented in the form $\varphi + \chi$, where φ is a single-valued function of the coordinates, and χ is a linear function of the principal circulations of the liquid in the multiply connected cavities. Let p, q, r be the components of the angular velocity of the body in a coordinate system rigidly attached to the body, the directions of the axes being chosen so that Ap, Bq, Cr represent the components, taken with respect to the fixed point, of the angular momentum of the body and of the angular momentum of the liquid corresponding to the potential φ. The constants λ, μ, ν determine the angular momentum of the liquid corresponding to the potential χ. Denote by e_1, e_2, e_3 the direction cosines of the ray going from the fixed point through the center of gravity of the body, and by $\gamma_1, \gamma_2, \gamma_3$ the components of a vector directed along the force of gravity and equal in magnitude to the product of the weight of the body by the length of the segment joining the center of gravity with the fixed point.

The equations of motion of the body ([1])

$$\begin{aligned} A\frac{dp}{dt} + (C-B)qr + \nu q - \mu r &= e_2\gamma_3 - e_3\gamma_2, \\ B\frac{dq}{dt} + (A-C)rp + \lambda r - \nu p &= e_3\gamma_1 - e_1\gamma_3, \\ C\frac{dr}{dt} + (B-A)pq + \mu p - \lambda q &= e_1\gamma_2 - e_2\gamma_1, \end{aligned} \tag{1}$$

$$\frac{d\gamma_1}{dt} = r\gamma_2 - q\gamma_3, \qquad \frac{d\gamma_2}{dt} = p\gamma_3 - r\gamma_1, \qquad \frac{d\gamma_3}{dt} = q\gamma_1 - p\gamma_2$$

have the known integrals

$$\frac{1}{2}(Ap^2 + Bq^2 + Cr^2) - (e_1\gamma_1 + e_2\gamma_2 + e_3\gamma_3) = h, \tag{2}$$

$$(Ap + \lambda)\gamma_1 + (Bq + \mu)\gamma_2 + (Cr + \nu)\gamma_3 = m, \tag{3}$$

$$\gamma_1^2 + \gamma_2^2 + \gamma_3^2 = \gamma^2. \tag{4}$$

With the aim of generalizing to the problem under consideration the well-known Bobylev-Steklov case of integrability ([2]), established for a body without liquid filling, we set

$$p = \text{const} = n, \qquad r = 0, \qquad \nu = 0, \qquad e_2 = e_3 = 0, \tag{5}$$

whereupon the first equation (1) becomes an identity, while the following give

$$B\frac{dq}{dt} = -\gamma_3, \tag{6}$$

$$\gamma_2 = [(B - A)n - \lambda]q + \mu n. \tag{7}$$

From integral (2) we find

$$\gamma_1 = \frac{B}{2}q^2 + H,$$

where $H = \frac{A}{2}n^2 - h.$

(8)

Substitute (5), (7), (8) into (3):

$$(An + \lambda)\left(\frac{B}{2}q^2 + H\right) + (Bq + \mu)\{[(B - A)n - \lambda]q + \mu n\} = m.$$

This relation, under the conditions

$$\lambda = (2B - A)n, \qquad \mu^2 n = m - 2BnH \tag{9}$$

becomes an identity. Determining from (4), (7), (8), (9)

$$\gamma_3 = \sqrt{\gamma^2 - (\mu - Bq)^2 n^2 - \left(H + \frac{B}{2} q^2\right)^2},$$

we find from (6) the dependence of q on t by inversion of the elliptic integral

$$t = -\int_{q_0}^{q} \frac{B\, dq}{\sqrt{\gamma^2 - (\mu - Bq)^2 n^2 - \left(H + \frac{B}{2} q^2\right)^2}},$$

as a result of which the dependence of $\gamma_1, \gamma_2, \gamma_3$ on t is also established.

Institute of Hydrodynamics
Siberian Branch of the Academy of Sciences of the USSR

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CITED LITERATURE

[1] N. E. Zhukovskii, *On the motion of a solid body having cavities filled with a homogeneous capillary liquid*, Collected Works, 3, Moscow-Leningrad, 1936. [2] D. Bobylev, Tr. Otd. Fiz. Nauk Obshch. Liubit. Estestvozn., 8, No. 2 (1896); V. Steklov, ibid.

Note: Figure translations are in progress. See original paper for figures.