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Abstract

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SAIDEM INSTABILITY FOR A FINITE ION LARMOR RADIUS

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In Suydam's paper ⁽¹⁾ it was shown that a plasma confined by a helical magnetic field $\mathbf{H} = (0, H_\varphi(r), H_z(r))$ is unstable with respect to perturbations of the form $f = f(r)e^{im\varphi+ikz-i\omega t}$, localized near a certain magnetic surface $r = r_0$, where

$$k_\tau(r_0) \equiv \left(\frac{m}{r}\frac{H_\varphi}{H} + k\frac{H_z}{H}\right)_{r=r_0} = 0,$$

if

$$\varkappa^2 \equiv -\left.\frac{8\pi p'}{rH_z^2}\left(\frac{\mu}{\mu'}\right)^2\right|_{r=r_0} > \frac{1}{4}. \qquad (1)$$

Here $\mu = H_\varphi/rH_z$; $p = n(T_i + T_e)$ is the equilibrium plasma pressure; n is the equilibrium density; T_i and T_e are the ion and electron temperatures; the prime denotes differentiation with respect to r.

This criterion was obtained in the magnetohydrodynamic approximation (MHD), when the perturbation frequency ω is considerably smaller than the ion cyclotron frequency $\Omega_i = eH/M_ic$, and the ion Larmor radius $\rho = \sqrt{T_i/M_i\Omega_i^2}$ is much smaller than the perturbation wavelength $\lambda \sim (m^2/r^2 + k^2)^{-1/2}$.

However, if the instability increments are sufficiently small, then the MHD approximation must be refined by taking into account terms small as $(\rho/\lambda)^2$. It is therefore of interest to determine how Suydam' s criterion changes if, following the ideas of the work of Rosenbluth et al. ⁽²⁾, one takes into account the finiteness of the ratio $(\rho/\lambda)^2$.

For this purpose, starting from the kinetic equation in the absence of collisions, we calculated the currents induced in a plasma of helical geometry by the field of an electromagnetic wave. It was shown that Maxwell' s system of equations can be reduced to the differential equation

$$\frac{d^2 E_n}{dy^2} - k_n^2 E_n + \frac{2y}{y^2 - b}\frac{dE_n}{dy} + \frac{\varkappa^2}{y^2 - b} E_n = 0. \tag{2}$$

Here the following notation has been introduced:

$$b = \frac{\omega^2}{v_A^2 (k_\tau')^2}\left(1 - \frac{k_n v_0}{\omega}\right),$$

$$k_\tau(r_0) = 0, \qquad k_\tau(r) = k_\tau'(r_0) y, \qquad y = r - r_0, \tag{3}$$

$$k_\tau' = -k_n r \mu' \left(\frac{H_z}{H}\right)^2, \qquad k_n = -\frac{m}{r}\frac{H}{H_z}, \qquad E_n = \frac{H_\varphi}{H} E_z - \frac{H_z}{H} E_\varphi,$$

$$v_0 = -\frac{1}{\Omega_i n M_i}\frac{d}{dr}(nT_i)$$

is the velocity of the ion Larmor drift,

$$v_A = \left(\frac{H^2}{4\pi n M_i}\right)^{1/2}$$

is the Alfvén velocity. All coefficients in equation (2) are taken at the point $r = r_0$, and the prime denotes the derivative with respect to r, taken at $r = r_0$.

Equation (2) is valid under the following assumptions: $\omega/\Omega_i \ll 1$; $(\rho/\lambda)^2 \ll 1$ (as in the MHD approximation); $8\pi p \ll H^2$ (the plasma pressure is small in comparison with the magnetic pressure); $k_\tau \ll k_n$ (the wave vector

almost perpendicular to the magnetic field); $\omega/k_n \ll v_A$ (the phase velocity of the wave is small compared with the Alfvén velocity), $\partial E/\partial r \gg E/r$ (the perturbations are strongly localized).

We note that equation (2) differs from the corresponding MHD equation only in that in the MHD approximation $k_n v_0 \simeq 0$, so that $b = \omega^2/v_A^2(k_r')^2$.

In the case $b < 0$ (it is precisely this case that corresponds to instability in the MHD approximation), passing to the new variables $F = E_n\sqrt{y^2 - b}$, $s^2 = -y^2/b$, one can reduce equation (2) to the form

$$\frac{d^2 F}{ds^2} + b k_n'^2 F + \left[\frac{\varkappa^2}{s^2 + 1} - \frac{1}{(s^2 + 1)^2}\right] F = 0. \tag{4}$$

Equation (4) is analogous to the Schrödinger equation for a particle whose energy is

$$\varepsilon = bk_n'^2 < 0, \tag{5}$$

and the potential is

$$U = -\frac{\varkappa^2}{s^2+1} + \frac{1}{(s^2+1)^2}. \tag{6}$$

The form of the potential U, and consequently also the eigenvalues of the energy $\varepsilon = \varepsilon_n$, are the same both in the MHD approximation and in our approximation, which takes into account the finiteness of $(k_n\rho)^2$.

From (3) and (5) we obtain

$$\omega = \frac{k_n v_0}{2} \pm \sqrt{\left(\frac{k_n v_0}{2}\right)^2 + \varepsilon_n \left(\frac{k_\tau' v_A}{k_n}\right)^2}, \tag{7}$$

which in the MHD approximation ($k_n v_0 \to 0$) goes over into the equation

$$\omega = \pm \frac{k_\tau' v_A}{k_n}\sqrt{\varepsilon_n}.$$

Thus, the MHD approximation gives that the system is unstable if there is a bound state with $\varepsilon_n < 0$. For $\varkappa^2 < 1/4$ the potential well does not have sufficient depth for a level to appear, and the plasma is stable (^{1,3}).

Let there be instability in the MHD approximation, i.e. $\varkappa^2 > 1/4$, and let the well be sufficiently deep for a level with $\varepsilon_n < 0$ to appear. When the finite Larmor radius of the ions is taken into account ($k_n v_0 \neq 0$), the plasma will nevertheless be locally stable if the expression under the radical in (7) is positive:

$$\left(\frac{k_n v_0}{2}\right)^2 > |\varepsilon_n| \left(\frac{k_\tau' v_A}{k_n}\right)^2. \tag{8}$$

Since $|\varepsilon_n|$ in any case does not exceed the magnitude of the minimum of the potential U, it follows from (6) and (8) that the plasma is stable even for large $\varkappa^2$, if the ion Larmor radius is sufficiently large, so that

$$\left(\frac{m}{r}\rho\right)^2 > \frac{a}{R}\left(1 + \frac{T_e}{T_i}\right). \tag{9}$$

Here a is the characteristic scale of the plasma inhomogeneity, $a^{-1} = d\ln p/dr$, R is the radius of curvature of the field line, $R = r(H/H_\varphi)^2$.

Thus, when the finite ion Larmor radius is taken into account, a stabilizing effect appears, the possibility of which was pointed out in (^{2}).

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Note: Figure translations are in progress. See original paper for figures.