Corresponding Member of the Academy of Sciences of the USSR G. G. CHERNYI

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Abstract

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HYDROMECHANICS

Corresponding Member of the Academy of Sciences of the USSR G. G. CHERNYI

ON THE ANALOGY WITH AN EXPLOSION IN HYPERSONIC FLOW PAST A THIN BODY BLUNTED AT THE FRONT*

Ten years ago the idea was put forward of the equivalence of the motion in a gas of a blunted body at hypersonic speed and of a strong explosion of a cylindrical charge ([1,2]). This idea was developed and confirmed by comparison with experimental data in works ([3–5]). The completion of the foundations of the theory following from the analogy with an explosion was the establishment of a similarity law for hypersonic flow past thin bodies blunted at the front ([6,7]). As one of the examples of the similarity established in ([6]) for various cases of flow past geometrically dissimilar bodies, Fig. 1 gives, in the original variables and in similarity variables, the pressure distributions on cones with different opening angles and different shapes of the nose part. The initial numerical data are taken from the tables of ([8]).

The analogy with a strong explosion made it possible not only to establish the similarity law. In combination with the concept of a strongly compressed layer of gas behind the shock wave, it served as the basis for a simple method of integral relations ([5]), by means of which the first problems on non-self-similar flows around blunted thin bodies were solved theoretically and an important qualitative difference was established in the behavior of the drag of a blunted profile and of a blunted body of revolution. This difference is due to the fact that, in flow past a wing profile, a small bluntness of its leading edge **raises** the pressure over a considerable part of the profile as compared with the pressure on a sharp profile, whereas, in flow past a body of revolution, bluntness of the front end **lowers** the pressure over a considerable part of the surface of such a body.

Figure 2 shows the variation of the drag coefficients of a blunted thin wedge and a blunted thin cone as functions of their length. The dashed line gives the values of the drag computed without taking into account the influence of bluntness on the pressure distribution over the remaining part of the body (as is done, for example, when using Newton’ s formula). The reverse mutual arrangement of the curves in the case of the wedge and in the case of the cone is evident. For characteristic cases of flow, the drag of a blunted cone can be very close to the drag of a sharp cone, whereas the drag of a blunted wedge in such cases

Figure 1

Figure 1: Figure 1

considerably exceeds the drag of a sharp wedge.

Since the analogy with a strong explosion is based on the additional hypothesis that it is possible to neglect the influence of the gas layer near the surface of the body in which the law of plane sections is not satisfied, it does not give *a priori* the true asymptotic behavior of the flow. An asymptotic theory, taking into account the presence of a layer with high entropy near the surface of the body, is being developed in works ([9]), but has not yet led to concrete results. Estimates made by a number of authors ([7,10,11]) for the case of inverse problems (in which the shape of the shock wave was prescribed) indicated a possible substantial influence of the layer with high entropy on the flow past blunted bodies. It was shown that, in the case of a parabolic shock wave $y^* = C_\nu x^{2/(\nu+2)}$

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($\nu = 1$ for plane flows, $\nu = 2$ for axisymmetric flows) the asymptotic expression for the shape of the body has the form $y_T = \bar{C}_\nu x^{2/(\nu+2)}$ (the constants C_ν and $\bar{C}_T$ are of the same order of magnitude), whereas, in accordance with the blast analogy, the shock wave mentioned should correspond, for $\nu = 1$, to a thin plate and, for $\nu = 2$, to a cylinder. Thus the inconsistency of the blast analogy for the asymptotic description of hypersonic flow past thin bodies blunted at the front was being demonstrated. In reality, however, the result obtained does not contradict the analogy with an explosion. We shall show this.

Fig. 1. $1 - \alpha = 5^\circ,\ M = \infty,\ \delta = 1$; $2 - \alpha = 10^\circ,\ M = \infty,\ \delta = 1.5$; $3 - \alpha = 10^\circ,\ M = \infty,\ \delta = 1$; $4 - \alpha = 10^\circ,\ M = \infty,\ \delta = 0.5$; $5 - \alpha = 20^\circ,\ M = \infty,\ \delta = 1$

Let us emphasize that the theory of flow past thin bodies blunted at the front applies, strictly speaking, only to such bodies whose entire surface can be divided into a small forward blunt part, where the angles between the surface and the direction of the incident flow are of order $\pi/2$, and the remaining part, where these angles are small. For such bodies the flow region can be divided into an "inner" entropy layer, in which the gas density is small, and an outer disturbed flow, in which, to a first approximation, the law of plane sections may be regarded as satisfied.

Taking the pressure p to be constant across the inner layer and neglecting in Bernoulli's integral the transverse component of velocity in comparison with the longitudinal one, from the continuity equation we find inside the layer ($r < r_s$, r is the Lagrangian coordinate)

Fig. 2

Figure 2: Fig. 2

$$y^\nu - y_T^\nu = \psi_p'(p, r), \tag{1}$$

where the "momentum-loss" function

$$\Psi(p, r) = \rho_1 V \int_0^r (V - u)\, dr^\nu$$

is determined by the shape of the shock wave near the blunting.

Condition (1) at $r = r_s$, together with the conditions on the shock wave, determines the flow outside the entropy layer. For a perfect gas with constant heat capacities, condition (1) can be rewritten in the form

$$y_T^\nu = \frac{y_s^\nu p_s^{1/\gamma} - \int_0^{r_s} \frac{V}{u} \rho_1^{1/\gamma} e^{\Delta S/c_p}\, dr^\nu}{p_s^{1/\gamma}}.$$

Fig. 2. 1—$c_x = 1,\ \alpha = 17^\circ,\ l/d = 5;\quad$ 2—$c_x = 1,\ \alpha = 9^\circ,\ l/d = 100$

We shall use, for describing the asymptotic behavior of the flow outside the entropy layer, the solution of the explosion problem. Then the first term in the numerator is determined by this asymptotic behavior, while the second term is determined by the shape of the shock wave near its vertex. Both of these terms are asymptotically constant; moreover

$$y_s^\nu p_s^{1/\gamma} \to [\chi(\gamma) E_{\mathrm{B}}]^{1/\gamma} r_s^{\nu(\gamma-1)/\gamma}. \tag{12}$$

If the energy E_{B} in this expression is taken equal to the value of the corresponding constant E characterizing the shape of the shock wave near the vertex, then the numerator of the expression for y_T^ν tends to a nonzero constant, and since in this case $p_s \sim x^{-2\nu/(\nu+2)}$, we obtain the result mentioned above, $y_T \sim x^{2\gamma/(\nu+2)}$. But the energy of the asymptotic flow E_{B} need not be equal to E and may be chosen precisely so that the two terms in the numerator coincide. Then, in the first approximation, $y_T = 0$; in order to find the outer flow corresponding to $y_T = \text{const} \neq 0$, one must introduce next-order correction terms to the flow from the explosion. In particular, if the shape of the shock wave near the vertex is parabolic and if, as was done in papers ([7], [10]), one assumes that r_s is determined by the condition $dy^*/dx = 1$, then, for flow past a plate or a cylinder with a blunting corresponding to a parabolic shock wave, we obtain the following relation between E_{B} and the constant E:

$$E_{\mathrm{B}} = \left(2^{(\gamma-1)/\gamma} - 1\right)^{\gamma} E.$$

Thus, the problem of hypersonic flow past a thin body blunted at the front, with allowance for the entropy layer, can asymptotically be regarded as the problem of a flow in a plane arising in an explosion with subsequent expansion of a piston. The solution of the corresponding equations must satisfy the conditions on the shock wave and the asymptotic condition as $r \to 0$ ($r \ll y^*$)

$$(y^{\nu} - y_T^{\nu}) p^{1/\gamma} = [\chi(\gamma) E_{\mathrm{B}}]^{1/\gamma} r^{\nu(\gamma-1)/\gamma}.$$

where

$$[\varkappa(\gamma) E_{\mathrm{B}}]^{1/\gamma} r_s^{\gamma(\gamma-1)/\gamma} = \int_1^{r_s} p_1^{1/\gamma} e^{\Delta s/c_p}\, dr^*.$$

In passing to the problem of the flow past a thin body, this solution should be used only for $r > r_s$.

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Note: Figure translations are in progress. See original paper for figures.