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Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Abstract

Full Text

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AN EXAMPLE OF A MAGNETIC-FIELD STRUCTURE WITH DESTRUCTIBLE MAGNETIC SURFACES

(Presented by Academician M. A. Leontovich, June 22, 1963)

In work ([1]), by means of a numerical calculation, it was shown that among the lines of force of the magnetic field ($\mathbf{H} = \nabla\Phi$)

$$\Phi = z + h_0 I_0(3r)\sin 3z + 3I_3(3r)\sin 3(\varphi - z),$$

which, at $z = 0$, issue from points of the positive semiaxis x (i.e. $\varphi = 0$), there are periodic lines of force, with period $2\pi N/3$, and for $h_0 = 0.125$ the quantity $N = 1, 12, 13, \ldots$. In work ([1]) the interval $0 < x < x_{O'}$ was investigated, where O' is the so-called center of a petal.

Fig. 1

Fig. 2

In the present work we investigate the behavior of lines of force lying in a small neighborhood of periodic lines of force. As in the preceding work, the character of the behavior of the lines of force is represented by means of the mapping plane ([1]), on which the periodic lines of force are represented by a system of N points (called rational in ([1])).

The first task of the present work consisted in determining the character of the behavior of the mapping points in a neighborhood of rational points.

For this purpose, in a sufficiently small neighborhood of each rational point $A(x_0, 0, N)$, determined approximately from the graph $\delta(x)$ ([1]), four points were chosen, arranged crosswise. From these points we launched lines of force, which we calculated over a large period

$$\frac{2\pi}{3}N,$$

with two lines of force calculated along the positive direction of the z-axis, and two along the negative direction of the z-axis. Knowing the new positions of the four points taken, one can refine the coordinate x_0 ($y_0 = 0$) of the rational point and determine the transformation matrix $(\alpha_{ik})_N$ ([2]), showing how, over the large period $\frac{2\pi}{3}N$, the coordinates $\xi = x - x_0$, $\eta = y - y_0$ of an arbitrary point located in a small neighborhood of the rational point change. The determinant of this matrix is equal to unity, $|\alpha_{ik}| = 1$ by virtue of the condition $\operatorname{div}\mathbf{H} = 0$, and, consequently, the proper values λ_1, λ_2 of the matrix $(\alpha_{ik})_N$ are related by $\lambda_1\lambda_2 = 1$. If λ is imaginary, then the rational point is elliptic, while if λ is real, the point is hyperbolic; moreover, for $\lambda > 0$ the mapping points move along hyperbolas without jumps, and for $\lambda < 0$ with jumps ([2]).

Using the method described, it was found that rational points with odd N, for any h_0, are always hyperbolic with $\lambda > 0$, while even rational points for $N < N^*$ are elliptic, and for $N > N^*$ are hyperbolic with $\lambda < 0$. Here N^* is the number of the point with coordinate x^* (see (1)).

Table 1

h_0	xO'	$N =$ 11	$N =$ 12	$N =$ 13	$N =$ 14	$N =$ 15	$N =$ 16	$N =$ 17	$N =$ 18	$N =$ 19	$N =$ 20
0,0120	0,07363				0,05838			0,04341			0,03510
0,0125	0,07630		0,06983	0,05783	0,05128	0,04649	0,04269	0,03956	0,03689	0,03461	0,03260
0,0130	0,07893	0,06938	0,05783					0,03637	0,03398	0,03234	0,03085

(continued)

h_0	xO'	$N =$ 21	$N =$ 22	$N =$ 23	$N =$ 24	$N =$ 25	$N =$ 26	$N =$ 27	$N =$ 28	$N =$ 29	$N =$ 30
0,0120	0,07363	0,03337	0,03163		0,02875	0,02735	0,02618	0,02511	0,02412	0,02322	0,02238
0,0125	0,07630	0,03084	0,02925	0,02784	0,02655	0,02539	0,02432*	0,02335	0,02244	0,02164	0,02084
0,0130	0,07893								0,02092	0,02018	0,01944

* The fixed limiting elliptic point on the left is marked. For $h_0 = 0,120$ this point corresponds to $N \approx 36$.

In Table 1 and in Figs. 1 and 2 are presented the coordinates of the rational points, the values $\lambda > 1$ and the angle α between the asymptotes for odd hyperbolic points, as well as the values of the semiaxis ratio b/a and the rotation parameter ω ($\lambda = e^{2\pi i\omega}$) for even elliptic points; also shown is the calculated

Fig. 4

Figure 3: Fig. 4

orientation of the asymptotes and ellipses corresponding to rational points lying in the interval $x^* < x < xO'$.

Fig. 3. a –hyperbolic points, b –elliptic points

From these data one can conclude that as $x \to x^*$ the ellipses flatten in the direction of the y-axis, $\omega \to 1/2$, whereas as $x \to xO'$ the ellipses elongate along the y-axis, the angle between the asymptotes tends to zero; moreover, $\lambda \to \lambda_{\text{min}} \approx 1$, $\omega \to \omega_{\text{min}}$.

In order to draw the picture in the xy-plane and thereby determine the interaction of the regions of different fixed points, we took the hyperbolic point $A\,(x_0, 0;\ 17)$ and, taking into account the stability of the asymptote,

moved along it. It turned out that this asymptote arrives at the point $A(x_1, y_1, 17)$, which is obtained from the point $A(x_0, 0, 17)$ as a result of displacement along the z-axis by a period equal to $2\pi/3$ ([1]).

Carrying out analogous calculations with $N = 16$, we obtained the picture shown in Fig. 3. In Fig. 3a the positions of the rational points corresponding to $N = 16$ and $N = 17$ are shown. In this figure it is not possible, at this scale, to show the structures arising around each rational point. However, it is seen that on each splitting magnetic surface with number N there are $2N$ alternating elliptic and hyperbolic points. The structure in the neighborhood of these points is shown in Fig. 3b, and in Fig. 4 a schematic general picture is given.

Fig. 4

It follows from this that the destruction (or, perhaps more accurately, the splitting) of magnetic surfaces begins on those surfaces on which the lines of force close after N turns. This destruction consists in the formation around one of the lines of force of a "fiber" which, if one period of the field is regarded as a torus, is laid N times on the former magnetic surface. In this case the "gluing" of neighboring turns of the "fiber" occurs along the line of force which is represented in the mapping plane by hyperbolic points. The mapping points of the line of force around which the fiber has formed are elliptic. The question of whether the surfaces forming the fiber are genuine surfaces or whether they also split remains unresolved, although the latter possibility seems more probable.

As is seen from Figs. 1 and 2, near the center of the lobe the splitting of the magnetic surfaces is weakly expressed, whereas as $N \to N^*$ the splitting increases. Thus we have the possibility of observing the dynamics of the splitting of magnetic surfaces on the example of a single field.

In particular, the dependence of the ratio of the semithicknesses of the fiber q

to the distance p between neighboring rational points lying on the x-axis proved to be of interest. For $N = 16, 17$ this ratio is equal to 0.096, while near $N^* = 26$, for $N = 20, 21$, it is equal to 0.30. In order to compare the dynamics of the splitting process thus obtained with that which occurs when h_0 is varied, α and λ were calculated for rational points lying at $x \simeq 0.0254$, for $h_0 = 0.120$ and $h_0 = 0.125$. It turned out that for $h_0 = 0.120$, in the neighborhood of the indicated point, $N = 27$, $\alpha = 1.18$ rad., $\lambda = 2.4$, whereas for $h_0 = 0.125$, $N = 25$, $\alpha = 1.66$, $\lambda = 4.8$ (cf. with ([1])). Thus, for a small change of h_0, there is a large change of α and λ, and consequently a large change in the thickness of the fiber.

In the region $N > N^*$ the elliptic points, as noted above, turn into hyperbolic ones with $\lambda < 0$. The character of the behavior at large distances of the asymptotes of these points proved to be very complicated and to require special investigation.

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Note: Figure translations are in progress. See original paper for figures.