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Academician of the Academy of Sciences of the BSSR B. I. STEPANOV

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Abstract

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ON THE STABILITY OF STATIONARY STATES OF QUANTUM GENERATORS

Let us consider the stationary states of a quantum generator of arbitrary form. Light rays pass inside the resonator along complex paths, being amplified as they traverse the thickness of the substance and attenuated upon reflection from the resonator surfaces. Under stationary conditions the amplification and attenuation of the flux compensate one another and therefore

$$r_1 r_2 r_3 \dots r_m \exp\left[-\int_0^L [k_{\text{st}}(s)+\rho(s)]\,ds\right] = 1. \tag{1}$$

Here L is the path traversed by the light over a large interval of time, including m reflections from the resonator walls. The reflection coefficients r_g from different points of the surface may be different. The parameter $\rho(s)$ characterizes energy losses due to scattering or absorption of light by impurities; the negative absorption coefficient $k_{\text{st}}(s)$ is its amplification. Introducing the mean values $\bar{k}_{\text{st}}$ and $\bar{\rho}$ along the path L, the integral in (1) may be replaced by the product $(\bar{k}_{\text{st}}+\bar{\rho})L$. Transforming (1), we obtain

$$-\bar{k}_{\text{st}} = \bar{\rho} + \frac{1}{L}\left[\ln\frac{1}{r_1} + \ln\frac{1}{r_2} + \dots + \ln\frac{1}{r_m}\right] = \bar{\rho} + \frac{1}{l}\ln\frac{1}{r} = k^{\text{loss}}, \tag{2}$$

where l is the mean path between two reflections, and $\ln(1/r)$ is the mean value of $\ln(1/r_g)$. It follows that the quantity $(1/l)\ln(1/r)$ characterizes the energy losses per unit path due to the escape of radiation beyond the resonator boundaries. For different rays $v, w, \dots$ propagating inside the layer in different directions, the mean energy losses upon reflection are different.

If all r_g are close to unity, then the values $\ln(1/r_g)$ are equal to $1-r_g = T_g + A_g$, where T_g and A_g are the transmission coefficient and absorptivity of the coating at the point of the g-th reflection. In this case formula (2) takes the obvious form

$$k^{\text{loss}} = -\bar{k}_{\text{st}} = \bar{\rho} + \frac{1}{L}\sum_{g=1}^{m} T_g + A_g = \bar{\rho} + \frac{T}{l} + \frac{A}{l}. \tag{2'}$$

For small r, in order to take account of reflection losses one must use formula (2). For large L, both the right- and left-hand sides of (2) do not depend on L.

The change in the density of the flux under consideration along the path $ds = \frac{c}{n}\,dt$ is determined by the Bouguer law ([1])

$$du_{iv} = [-k_{iv}u_{iv} - k_{iv}^{\text{loss}}u_{iv}]\,\frac{c}{n}\,dt. \tag{3}$$

The absorption coefficient k_{iv} at large radiation densities depends, in the general case, both on the radiation density u_{iv} with frequency ν_i and the direction of propagation of the flux v, and on the densities of other

of the fluxes propagating inside the resonator*. In the case of gases this dependence can be approximated by formula (2)

$$k_{iv} = k_{0i}(\nu_i)/\left[1 + \sum_i \alpha_i(\nu_i)u_i(\nu_i)\right], \tag{4}$$

where $k_{0i}(\nu_i)$ is the zero absorption coefficient, depending on the pumping; $\alpha_i(\nu_i)$ are the nonlinearity parameters, and $u_i(\nu_i) = \sum_v u_{iv}$ is the radiation density of frequency ν_i, summed over rays of all directions. Formula (4) can also be used, to a good approximation, for solids. If the value $-k_{iv}$ in (3) exceeds the loss value k_{iv}^{loss}, then the density u_{iv} increases with time. For $-k_{iv} - k_{iv}^{\text{loss}} < 0$ it decreases. The stationary regime is established under condition (2).

According to (2) and (4), stationary generation can exist only for those fluxes, or their combinations, for which

$$[-k_{0i} - k_{iv}^{\text{loss}}]/k_{iv}^{\text{loss}} = [-k_{0j} - k_{jv}^{\text{loss}}]/k_{jv}^{\text{loss}} = \cdots = \sum_i \alpha_i u_i > 0. \tag{5}$$

If it is known that, in the stationary regime, generation of only one type (i, v) occurred, then the radiation density is determined uniquely:

$$u_{iv} = [-k_{0i} - k_{iv}^{\text{loss}}]/\alpha_i k_{iv}^{\text{loss}} = \left[-k_{0i} - \rho - \frac{1}{l}\ln\frac{1}{r}\right]\Big/\alpha_i\left(\rho + \frac{1}{l}\ln\frac{1}{r}\right). \tag{6}$$

The amount of energy lost per unit volume in 1 sec is: due to absorption by impurities

$$W_{\text{in}}^{\text{loss}} = \frac{c}{n}u_{iv}\rho = \frac{c}{n}\frac{\left(-k_{0i} - \rho - \frac{1}{l}\ln\frac{1}{r}\right)\rho}{\alpha_i\left(\rho + \frac{1}{l}\ln\frac{1}{r}\right)} \simeq$$

$$\simeq \frac{\left(-k_{0i} - \rho - \frac{T}{l} - \frac{A}{l}\right)}{\alpha_i}\,\frac{\rho}{\rho + \frac{T}{l} + \frac{A}{l}}; \tag{7}$$

due to absorption in the coatings

$$W^{\text{loss}}_{\text{coat}} = \frac{c}{n} u_{iv} \frac{1}{l} \ln \frac{1}{r} \frac{A}{A+T} = \frac{c}{n} \frac{\left(-k_{0i} - \rho - \frac{1}{l} \ln \frac{1}{r}\right) \frac{1}{l} \ln \frac{1}{r} \cdot A}{\alpha_i \left(\rho + \frac{1}{l} \ln \frac{1}{r}\right)(A+T)} \simeq$$

$$\simeq \frac{c}{n} \frac{\left(-k_{0i} - \rho - \frac{T}{l} - \frac{A}{l}\right)}{\alpha_i \left(\rho + \frac{T}{l} + \frac{A}{l}\right)} \frac{A}{l}; \tag{8}$$

due to the emission of radiation beyond the coatings

$$W^{\text{em}} = \frac{c}{n} u_{iv} \frac{1}{l} \ln \frac{1}{r} \frac{T}{A+T} = \frac{c}{n} \frac{\left(-k_{0i} - \rho - \frac{1}{l} \ln \frac{1}{r}\right) \frac{1}{l} \ln \frac{1}{r} \cdot T}{\alpha_i \left(\rho + \frac{1}{l} \ln \frac{1}{r}\right)(A+T)} \simeq$$

$$\simeq \frac{c}{n} \frac{\left(-k_{0i} - \rho - \frac{T}{l} - \frac{A}{l}\right)}{\alpha_i \left(\rho + \frac{T}{l} + \frac{A}{l}\right)} \frac{T}{l}. \tag{9}$$

The maximum emission of the generator W^{em} is attained at (see (3))

$$T_{\text{m}} = -(\rho l + A) + \sqrt{-k_{0i} l(\rho l + A)}. \tag{10}$$

The value T_{m} is equal to zero in the absence of harmful losses inside:

* In symmetric resonators, individual rays sometimes do not intersect with other rays on any path L. In these

and in the generator coatings. Measurement of T_m makes it possible to determine the magnitude of the losses. The maximum outcoupling is equal to

$$W^{\text{out}}_m = \left[\sqrt{k_{0i}} - \sqrt{\rho + A/l}\right]^2. \tag{11}$$

If, in the stationary state, there exists simultaneously not only one field (iv), but also other fields (jw), ..., satisfying condition (5), then the radiation densities $u_{iv}, u_{jw}, \ldots$ cannot be determined uniquely; for given $k_{0i}, k_{0j}, \ldots$ and $k^{\text{loss}}_{iv}, k^{\text{loss}}_{jw}, \ldots$, only the sum $\sum_i \alpha_i u_i$ is known. The distribution of intensities depends on the conditions under which the given stationary regime arises. Condition (5) restricts the number of possible stationary states of the generator, although it still remains very large. Which of the possible states are realized in actuality can be determined by solving the system of equations (3), taking (4) into account.

Let us write out from system (3) two equations for the densities u_{iv} and u_{jw}:

$$-\frac{1}{k_{0j}} \frac{du_{jw}}{u_{jw}} = \left[\frac{1}{1 + \sum_i \alpha_i u_i} + \frac{k^{\text{loss}}_{jw}}{k_{0j}}\right] \frac{c}{n}\, dt;$$

$$-\frac{1}{k_{0i}}\frac{du_{iv}}{u_{iv}} = \left[\frac{1}{1+\sum_i \alpha_i u_i} + \frac{k_{iv}^{\text{loss}}}{k_{0i}}\right]\frac{c}{n}\,dt. \tag{12}$$

Subtracting the second equation (12) from the first and integrating, we obtain

$$[u_{jw}(t)/u_{jw}(0)]^{1/-k_{0j}} = [u_{iv}(t)/u_{iv}(0)]^{1/-k_{0i}} \exp\left\{-\left[k_{jw}^{\text{loss}}/(-k_{0j}) - k_{iv}^{\text{loss}}/(-k_{0i})\right]\frac{c}{n}t\right\}. \tag{13}$$

If

$$k_{jw}^{\text{loss}}/(-k_{0j}) > k_{iv}^{\text{loss}}/(-k_{0i}), \tag{14}$$

then, as $t \to \infty$, the value u_{jw} tends to zero. In the established stationary state, a field of type jw will be absent. Analogous reasoning is applicable to any pairs of possible fields. Those fields for which the values

$$k_{jw}^{\text{loss}}/(-k_{0j}) = k_{iv}^{\text{loss}}/(-k_{0i}) \tag{15}$$

have a minimum will coexist with one another, and moreover

$$[u_{jw}^{\text{st}}/u_{jw}(0)]^{1/-k_{0j}} = [u_{iv}^{\text{st}}/u_{iv}(0)]^{1/-k_{0i}}. \tag{16}$$

Thus, among the possible stationary states of the field, only those are stable for which the ratio of losses to the zero-signal gain coefficient, $-k_{0i}$, is minimal. All the remaining fields disappear with time, being suppressed by more stable states.

The arguments given are valid if at $t = 0$ at least one of the stable fields was present inside the resonator. If, however, $u_{iv}(0) = 0$, then other, less stable fields will be present inside the resonator. Nevertheless, the appearance of even traces of one of the stable fields (for example, as a result of luminescence) will inevitably lead to the destruction of all unstable states of the generator.

In studying the properties of the generating object, the principal problem reduces to finding the stable states, and there is no special need to determine all possible stationary states. The number of stable states is, as a rule, relatively small. To determine them it is not necessary to solve the complicated problem of standing waves inside a resonator of complex shape; it is sufficient to consider the paths of individual rays and to compare, for them, the magnitude of the losses with the value $k_{0i}(\nu_i)$ over a long time interval.

It is not difficult to draw, for example, rather general conclusions about the frequency of stable generation. If the reflection coefficients at all boundaries do not have appreciable spectral selectivity, then the losses associated with the output of the radiation are almost identical for all frequencies. At the same time, the zero-signal absorption coefficient k_{0i} depends strongly on frequency. Therefore generation

stable for those frequencies which correspond to the maximum of $|k_{0i}|$, i.e., to the maximum of the absorption band in the absence of generation. The interference restrictions (4) can somewhat change this frequency. If the coatings have a very narrow pass band, then the frequency of the stable field inside the generator will be close to the frequency for which the reflection of the coatings has the maximum value.

As a rule, all stable fields have one and the same frequency, independently of the direction of propagation of the fluxes. In view of this, the parameter α in formulas (4) and (5) is the same, which makes it possible to determine the mean radiation density inside the resonator:

$$u = \sum_{i\nu} u_{i\nu} = \frac{k_0(\nu_{\max}) - k_{\min}^{\text{loss}}}{\alpha(\nu_{\max}) - k_{\min}^{\text{loss}}}. \tag{17}$$

In accordance with (14) or (15), in the stable state of the generator the radiation density is maximal. Comparison of (17) with (6) shows that it does not depend on the number of stable fluxes. An increase of one flux inevitably leads to a decrease of the others.

The directions of propagation of the stable fluxes are connected with the geometry of the resonator. Consider, for example, an unbounded plane-parallel layer. The minimum value of the losses is realized for fluxes incident on the bases of the layer at an angle $\theta > \theta^*$, where θ^* is the angle of total reflection. The value of u is determined by formula (6) for $r = 1$. For $r = 1$ the radiation does not leave the layer; within the layer such a radiation density u is established that the amplification of the fluxes over the path L is compensated by their absorption or scattering. For small ρ the value of u is very large.

Analogous arguments are valid for a spherical resonator. The most stable fluxes are those moving along chords close to the surface of the sphere and not emerging outside because of total reflection.

For a bounded plane-parallel layer it is necessary to take into account radiation losses at the lateral surfaces. In a layer bounded by a cylindrical surface, there are rays, parallel to the bases, which propagate along chords and undergo total reflection at the lateral boundaries. They are the most stable, although useless, since the pump energy is lost inside the layer. Such rays do not enter the region close to the axis of the resonator. In this region there will propagate fluxes almost parallel to the axis of the cylinder and determining the real generation of the layer.

In the general case the calculation of ray losses is complicated. It is not difficult to consider only the course of rays in the principal section of the cylinder. If a ray propagates at an angle θ to the axis, then over the path L the number of reflections from the base of the layer is equal to $\frac{L}{l}\cos\theta$, and the number of reflections at the lateral surfaces is $\frac{L}{d}\sin\theta$, where l is the height and d the diameter. The total losses upon reflection, calculated per unit path, are equal to

$$k^{\text{loss}} = \frac{\cos\theta}{l}\ln\frac{1}{R_\theta} + \frac{\sin\theta}{d}\ln\frac{1}{r_\theta},$$

where R_θ and r_θ are the reflection coefficients at the bases and the lateral surfaces. As a rule, near $\theta = 0$, as θ increases the losses decrease. However, if reflection at the lateral walls is absent, then generation of the layer is strictly directed. A detailed study of such resonators with allowance for the features of the coatings will be carried out subsequently.

Institute of Physics
Academy of Sciences of the USSR

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Note: Figure translations are in progress. See original paper for figures.