A. L. RABINOVICH, A. V. TURAZYAN

1963

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

PHYSICAL CHEMISTRY

A. L. RABINOVICH, A. V. TURAZYAN

THE INFLUENCE OF DEFORMATION RATE ON THE MAGNITUDE OF DEFORMATION AND THE STRENGTH OF ORIENTED GLASS-REINFORCED PLASTICS

(Presented by Academician A. V. Topchiev, August 16, 1962)

In uniaxial tension or compression of solid bodies (metals, polymers) at a constant rate of relative deformation ($v_\varepsilon = \text{const}$), the value of v_ε affects the maximum stress (the ultimate strength σ_b) and even the character of the relation between the stress σ and the deformation ε during tension (see, for example, ([1–3])). The dependence $\sigma_b = \sigma_b(v_\varepsilon)$ for metals was discovered as early as by Ludwik, who in ([4]) gave the empirical formula $\sigma_b = \sigma_1 + \sigma_0 \ln(v_\varepsilon / v_{\varepsilon,0})$. In ([1,2]) it was noted that, for metals, the relation between $\sigma_{\max} = \sigma_b$ and the rate of steady deformation is the same in the regime $v_\varepsilon = \text{const}$ and $\sigma = \text{const}$ (creep) —this opens up the possibility of determining creep parameters in short-time tests. However, the dependence of σ_b on v_ε in metals is comparatively weak. The influence of v_ε on the strength of polymers is considerably more noticeable. (For experimental data for homogeneous polymers see ([5–7]).)

Fig. 1

Glass-reinforced plastics are composite materials; they consist of glass fibers joined by a polymer binder into a single system. The properties of the polymers also predetermine certain features of the mechanical behavior of the entire system as a whole. A theoretical treatment of the purely mechanical compatibility of deformation of reinforcing fibers and binder in the case of a plane stress state is given in ([8]), where the reinforcing elements were assumed to be ideally elastic, and in the polymer binder, on the basis of works ([5,8–12]), only two of the three components of deformation were taken into account—elastic and highly elastic; residual deformation was neglected. To express the rate of highly elastic deformation of a structurally stable polymer, the Maxwell–Gurevich equations ([10]) were used; these are quite satisfactorily confirmed in the case of uniaxial tension ([2,12]).

For isothermal uniaxial tension (compression) of a glass-reinforced plastic whose axes of elastic symmetry are situated at an angle α to the direction of tension (Fig. 1), from the general equations ([8]) we obtain the following approximate relations:

$$\varepsilon_x = e_x + \varepsilon_x^*, \qquad e_x = \frac{\sigma_x}{E_{xx}}, \qquad \frac{\partial \varepsilon_x^*}{\partial t} = \frac{f_x}{\eta_x} \exp\left|\frac{f_x}{m_x}\right|, \tag{1}$$

where $f_x = \sigma_x - E_{\infty,x}\varepsilon_x^* = [1 + (E_{\infty,x}/E_{xx})]\sigma_x - E_{\infty,x}\varepsilon_x$. The notation used is as follows: t is time; $\varepsilon_x(t, x)$ is the total deformation; $e_x(x, t)$, $\varepsilon_x^*(x, t)$ are the elastic and highly elastic components; E_{xx} is the modulus of elasticity in the given direction; $E_{\infty,x}$, m_x, and η_x are respectively the modulus of highly elastic deformation, the rate modulus, and the coefficient of initial viscosity of this deformation. The last three parameters, as well as E_{xx}, de-

depend on α; their values can either be calculated from the known properties of the components of the fiberglass (fiber, binder), or determined directly from an experiment with the fiberglass. It should be kept in mind that the parameters m_x and η_x depend substantially on the temperature ϑ°K, the first almost linearly and the second approximately exponentially.

The solution of equations (1) together with the dynamic ones depends on the loading or deformation regime, i.e., on specifying on the surface of the body the functions $\sigma = \sigma(t)$ or $\varepsilon = \varepsilon(t)$.

Under conditions of homogeneous deformation in tension at a constant rate $v_\varepsilon = d\varepsilon/dt = \text{const}$, from (1) we obtain the differential equation of the tensile (compressive) diagram:

$$\frac{1}{E_{xx}} \frac{d\sigma_x}{d\varepsilon_x} = 1 - \frac{\varphi_x}{1 + (E_{\infty,x}/E_{xx})} \exp\left[-\frac{f_{x,\max}(1 - \varphi_x)}{m_x}\right], \tag{2}$$

where $\varphi_x = f_x/f_{x,\max}$, $f_{x,\max} = (f_x)_{\varepsilon\to\infty} \simeq [1 + (E_{\infty,x}/E_{xx})]\sigma_{x,\max} - E_{\infty,x}\varepsilon_{x,\max}$.

The last expression for $f_{x,\max}$ is valid under the assumption that the instant of failure is the completion of the deformation process and that the point ($\sigma_{x,\max} = \sigma_b$, $\varepsilon_{x,\max}$) belongs to the diagram $\sigma_x = \sigma_x(\varepsilon_x)$ obtained from (2).

The integral curve determined by equation (2) has the same form as for a homogeneous polymer, for which it is given in work ([12]); the only difference is that here the parameters $E_{xx}, \ldots$ depend on α. In ([12]) it is shown that the function $f_x(\sigma_x, \varepsilon_x)$ cannot increase without bound; it reaches a maximum value $f_{x,\max}$. The equation $\dot{f}_x = f_{x,\max}$ determines the asymptote of the integral curve. The value $f_{x,\max}$ is determined from the equation

$$\psi(\xi) \equiv \xi \exp \xi = b, \tag{3}$$

where $\xi = f_{x,\max}/m_x$, $b = \eta_x v_x/[1 + (E_{\infty,x}/E_{xx})]\, m_x$. The function $\psi(\xi) = \xi \exp \xi$ has been tabulated; therefore, when the known values m_x, b are available, the root of equation (3) is found directly by numerical selection.

If $f_{x,\max}$ is replaced by its approximate expression in terms of $\sigma_{x,\max}$ (2), then equation (3) may be regarded as an equation relating $\sigma_{x,\max} = \sigma_b$ and v_ε. In this case the quantity $\varepsilon_{x,\max}$ remains undetermined. However, experiment shows that $\varepsilon_{x,\max}$ is practically independent of v_ε (see Table 1).

Taking this fact into account and choosing as the basis for comparison some conventional standard regime, for example $v_{\varepsilon,0} = 1\%/\text{min}$, we obtain from (3), after taking logarithms,

$$\sigma_b - \sigma_b^0 = \frac{m_x \ln 10}{1 + E_{\infty,x}/E_{xx}} \left\{ \lg\left(\frac{v_\varepsilon}{v_{\varepsilon,0}}\right) - \lg\left[\frac{\sigma_b - E^*_{\infty,x}\varepsilon_{x,\max}}{\sigma_b^0 - E^*_{\infty,x}\varepsilon_{x,\max}}\right]\right\}. \tag{4}$$

Here the notation has been used for the reduced (“tangent”) modulus

$$E^*_{\infty,x} = E_{\infty,x}/(1 + E_{\infty,x}/E_{xx})$$

and for the stress corresponding to the standard regime

$$\sigma_b^0 = (\sigma_b)_{v_{\varepsilon,0}}.$$

In the region of not too small σ_b, for example $\sigma_b/\sigma_b^0 > 0.2$, the second term inside the braces in (4) is very small in comparison with the first, and for an approximate estimate of the effect of rate on the strength limit one may use the “linear” formula

$$\sigma_b - \sigma_b^0 = \frac{1}{1 + E_{\infty,x}/E_{xx}}\, m_x \ln 10 \,\lg\left(\frac{v_\varepsilon}{v_{\varepsilon,0}}\right). \tag{5}$$

Verification of the theoretical results was carried out on the basis of experimental data obtained in compression of SVAM specimens on a butvar-phenolic binder (BF-4). Specimens measuring 15 × 15 × 22 mm were cut from a plate 16 mm thick, made from non-standard

fiberglass yarn.* The tests were carried out to failure at a constant rate of strain, the magnitude of which was different for each of the four groups of specimens, each containing 5–6 identical samples.

Strain was measured with dial indicators, which determined the change in the total length of the specimen. The readings of the indicators, the force-measuring scale, and the electric clocks were recorded visually at low speed, and at high speed by motion-picture filming** with Konvas cameras at a speed of 16–24 frames per second. The accuracy of measurement thereby obtained was quite

Fig. 2

Figure 2: Fig. 2

sufficient for determining $\sigma_{x,\max}$ and the strain rate v_ε. The latter was determined from the diagram $\varepsilon_x = \varepsilon_x(t)$, constructed on the basis of instrument readings for each specimen.

Fig. 2

The test results are presented in Table 1 and in Fig. 2, where the average values for each group of specimens are given; deviations of individual values from the averages did not exceed $\pm 10\%$. For comparison, the table gives data for duralumin AK-8 obtained in ([2]).

As can be seen from Table 1, changing the strain rate over wide limits—by more than 4 orders of magnitude—practically does not lead to a change in the maximum strain. The value $\varepsilon_{x,\max}$ proves to be a very stable material characteristic.

Table 1

Compression of SVAM, $\alpha = 0^\circ$	Compression of SVAM, $\alpha = 0^\circ$	Compression of SVAM, $\alpha = 0^\circ$	Compression of SVAM, $\alpha = 45^\circ$	Compression of SVAM, $\alpha = 45^\circ$	Compression of SVAM, $\alpha = 45^\circ$	Duralumin (tension)	Duralumin (tension)	Duralumin (tension)
v_ε %/min	σ_b kg/cm^2	$\varepsilon_{x,\max}$ %	v_ε %/min	σ_b kg/cm^2	$\varepsilon_{x,\max}$ %	v_ε %/min	σ_b kg/cm^2	$\varepsilon_{x,\max}$ %
0.02	2880	3.3	–	–	–	0.02	5100	12.4
1.05	3520	2.8	2.9	790	6.5	8.5	5450	14.8
112	4220	2.4	–	–	–	–	–	–
234	4330	2.6	355	1055	7.1	–	–	–
–	–	–	–	–	–	6000	6800	11.6

The experimental dependence $\sigma_b = \sigma_b(v_\varepsilon)$ for $\alpha = 0^\circ$ in semilogarithmic coordinates $(\sigma_b, \lg v_\varepsilon)$, shown in Fig. 2, is practically linear, which confirms the applicability of the approximate formula (5). On the basis of this formula, the tangent of the inclination angle of the experimental straight line in Fig. 2 makes it easy to determine the rate modulus m_x, since

$$\frac{\Delta \sigma_b}{\Delta \lg \left(v_\varepsilon / v_{\varepsilon,0} \right)} \simeq \frac{\ln 10}{1 + \left(E_{\infty,x} / E_{xx} \right)}\, m_x. \tag{6}$$

Let us note that often $(E_{\infty,x}/E_{xx}) \ll 1$, and formula (5) can be replaced by an even simpler one:

$$\sigma_b - \sigma_b^0 \simeq m_x \ln 10 \lg\left(v_\varepsilon / v_{\varepsilon,0}\right). \tag{7}$$

* For this reason, the "standard" strength limit σ_b^0 in our experiments proved to be somewhat lower than the optimum value indicated in the work of the authors of SVAM ([12]).

** The motion-picture filming was performed by M. A. Uvarov, to whom the authors express their gratitude.

In this case the quantity ($m_x \ln 10$) gives the change in the ultimate strength corresponding to a change in the ratio of rates by one order of magnitude.

The value of m_x for an equal-strength GFRP, on the basis of the experiments described, proved to be 173 kg/cm^2 at $\alpha = 0°$ and 55 kg/cm^2 at $\alpha = 45°$. In this case, a change in the strain rate by one order of magnitude ($v_{\varepsilon_0} = 1\%/\text{min}$) leads to a relative change in the ultimate strength of the GFRP (σ/σ_b^0) of $\sim 10\%$ for $\alpha = 0°$ and $\sim 15\%$ for $\alpha = 45°$. For comparison, we note that analogous values for AK-8 duralumin are $\sim 6\%$, and for cold-worked copper $\sim 3\%$ ([2]). The figures cited indicate a noticeably greater influence of strain rate on the strength of glass-reinforced plastics than on the strength of metals.

If in (1) we set $\sigma_x = \text{const}$, then we arrive at the creep equation. Analysis of this equation, taking into account the independence of $\varepsilon_{x,\max}$ from σ, shows that under certain conditions it can yield a formula for the relation between the applied stress and the time of its action up to failure, analogous to the well-known formula of S. N. Zhurkov ([14]). It is important to note that the parameters contained in the indicated formula can be determined from short-term test data.

Institute of Chemical Physics
Academy of Sciences of the USSR

Received
16 VIII 1962

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Note: Figure translations are in progress. See original paper for figures.