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Abstract

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ON THE THEORY OF THE PROPAGATION OF CRYSTALLIZATION IN SUPERCOOLED CLOUDS

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Existing theories and methods for calculating the crystallization of supercooled clouds when they are seeded with ice nuclei, as set forth in works ($^{1-7}$), do not take into account the fallout of ice crystals from the cloud as they grow. Therefore they apply to the initial period of cloud dispersion, corresponding to the expansion of the crystallized zone, when the number of fallen ice crystals is small compared with the number of crystals that remain suspended. The solution of the full problem of the propagation of crystallization in supercooled clouds, taking into account the fallout of crystals, appears to be very laborious. In this connection, the present work proposes an approximate theory of the propagation of crystallization in supercooled clouds that takes into account the gradual fallout of ice crystals from the cloud and is therefore applicable to the entire period of propagation of crystallization in clouds.

As was shown in works (^{3,5,7}), the process of crystallization of clouds occurs practically in a narrow boundary, or frontal, zone separating regions with the crystalline and liquid phases. A diagram of the frontal zone is given in Fig. 1. The frontal zone is the region in which ice nuclei and droplets meet. In it, part of the nuclei grow to the size of falling particles, while the droplets evaporate completely. Droplets evaporate in a medium of ice crystals over the course of several seconds, and crystals in a medium of water droplets grow to the size of falling particles (10^{-3}—10^{-2} cm) over the course of several minutes. Thus, on the one hand, crystals cannot penetrate deeply into the region of the liquid phase, while on the other hand droplets cannot penetrate into the region of the crystalline phase. This circumstance is one of the reasons for the existence of sharp boundaries of the frontal zone. As crystalline nuclei enter the frontal zone, the excess moisture in it is eliminated, and a moment arrives when the space occupied by the zone is filled with nuclei that practically no longer grow, while the zone itself shifts toward the liquid phase. This process proceeds continuously.

Fig. 1. Diagram of the frontal zone in a crystallizing cloud layer (vertical section across the frontal zone)

In the work set forth here, we are primarily interested in the case of a long

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duration of crystallization propagation. In this case the transverse size of the zone l is small in comparison with the distance over which crystallization propagates, and the time of crystallization in it is small in comparison with the time of the entire process of crystallization of the cloud mass. In this connection, we shall regard the process in the frontal zone as spatially homogeneous. We shall further assume that the concentration of droplets to the left of plane A (Fig. 1) and the concentration of crystals to the right of plane B are equal to zero, that the concentration of vapor in the zone is equal to u, and that the concentration of crystals and vapor is un-

respectively near plane A on the left are n_2 and u_2, while the concentration of droplets and vapor immediately near plane B on the right are respectively n_1 and u_1. If the scale of the zone is l, and the coefficient of turbulent diffusion for the scale of the zone is k_l, then the number of droplets entering the zone from the region of the liquid phase per unit time, calculated per unit volume, may be estimated as $k_l \frac{n_1}{l^2}$. The corresponding number of crystals entering the zone from the region of the crystalline phase is equal to $k_l \frac{n_2}{l^2}$. On the basis of the results of [8], the system of equations describing the crystallization process in the zone, under the indicated simplified formulation of the problem, may be represented in the form

$$\frac{du}{d\tau} = -\frac{k_l}{l^2}(u-u_2)+\frac{k_l}{l^2}(u_1-u)+4\pi D n_1 (u_1-u) r_1^0 - 4\pi D \frac{n_2}{l^2} k_l (u-u_2) \int_{\varphi_2(\tau)}^{\tau} r_2 \, d\tau_2 + 4\pi D \frac{n_1}{l^2} k_l (u_1-u) \int_{\varphi_1(\tau)}^{\tau} r_1 \, d\tau_1; \quad (1)$$

$$r_1^0 = \left\{ R_1^2 - \frac{2D}{\rho_1}\xi \right\}^{1/2}; \quad (2)$$

$$\xi = \int_0^{\tau} (u_1 - u) \, d\tau'; \quad (3)$$

$$r_1 = \left\{ R_1^2 - \frac{2D}{\rho_1}(\xi - \xi_1) \right\}^{1/2}; \quad (4)$$

$$r_2 = \left\{ \frac{2D}{\rho^2}(\zeta - \zeta_1) \right\}^{1/2}, \qquad \zeta = \int_0^{\tau} (u - u_2) \, d\tau^1; \quad (5)$$

$$\varphi_1 \equiv 0 \quad \text{when } \xi \leqslant \frac{\rho_1 R_1^2}{2D},$$

$$\xi(\varphi_1) = \xi(\tau) - \frac{\rho R_1^2}{2D} \quad \text{when } \xi > \frac{\rho_1 R_1^2}{2D}; \tag{6}$$

$$\varphi_2 \equiv 0 \quad \text{when } \zeta \leqslant \frac{\rho_2 R_1^2}{2D},$$

$$\zeta(\varphi_2) = \zeta(\tau) - \frac{\rho_2 R_2^2}{2D} \quad \text{when } \zeta > \frac{\rho_2 R_2^2}{2D}. \tag{7}$$

In addition to the notation indicated above, in system (1)–(7) the following are denoted: by r_1^0, the radii of droplets that have been in the zone since the beginning of the process in it; r_1, the radii of droplets brought into the zone from the region of the liquid phase; r_2, the radii of crystals brought into the zone from the region of the crystalline phase; R_1, the initial radii of droplets; R_2, the radii of the precipitating crystals. The integration limits φ_1 and φ_2 in the right-hand side of equation (1) are determined by conditions (6), (7), which take into account the complete evaporation of droplets and the precipitation of crystals as they reach the sizes of precipitating particles.

Equation (1) describes the vapor balance in the frontal zone, referred to unit volume. In its right-hand side, the first and fourth terms describe the expenditure of vapor caused by its removal into the region of the crystalline phase and by crystal growth, while the second, third, and fifth terms describe the influx of vapor caused, respectively, by the inflow of vapor from the region of the liquid phase; by evaporation of droplets initially present in the zone; and by droplets that have entered the zone from the region of the liquid phase.

Practically complete crystallization of the frontal zone is accomplished in the case where the terms in the right-hand side of equation (1) describing-

the consumption of vapor, prevail over the terms accounting for its supply, and the derivative $du/d\tau$ satisfies the inequality $du/d\tau < 0$. Only in this case does the zone become practically completely crystallized, and the crystals can penetrate farther into the region of the liquid phase. Thus, the inequality $du/d\tau < 0$ is the condition ensuring the propagation of crystallization in a supercooled cloud mass.

Let us denote, in order, the terms on the right-hand side of equation (1) by A, B, C, D, and E. Then, for the inequality $du/d\tau < 0$ to be satisfied, it is necessary that the inequality

$$D + A > B + C + E. \tag{8}$$

Fig. 2. Distribution of the concentration of ice embryos $n_2(x, \tau)$, propagating in a crystallizing cloud mass (vertical section along the direction of propagation of crystallization). Diagram labels: n_2, $n_2(x, 0)$, $n_2(x, \tau)$, n_2^*, "liquid phase," "frontal zone," 0, x.

Figure 2: Fig. 2. Distribution of the concentration of ice embryos $n_2(x, \tau)$, propagating in a crystallizing cloud mass (vertical section along the direction of propagation of crystallization). Diagram labels: n_2, $n_2(x, 0)$, $n_2(x, \tau)$, n_2^*, "liquid phase," "frontal zone," 0, x.

be satisfied.

On the basis of the qualitative analysis carried out for the system (1)–(7), it was possible to show that, for the case in which crystals in the zone grow to the sizes at which they fall out and the droplets evaporate completely (such a regime in the zone always sets in, beginning with sufficiently small n_2), the inequalities

$$D \leq \frac{4}{3}\pi\rho_2 R_2^3 c n_2, \qquad c = \frac{k_1}{l^2}; \tag{9}$$

$$E \geq \frac{4}{3}\pi\rho_1 R_1^3 c n_1. \tag{10}$$

hold.

The equality signs in (9) and (10) correspond to crystal growth and droplet evaporation under conditions of constant supersaturation. In connection with the fact that, as crystallization proceeds, the difference $(u_1 - u)$ increases, the lifetime of the droplets decreases and inequality (10) becomes ever closer to equality. Therefore, in the course of the process the condition begins to be fulfilled

$$E \simeq \frac{4}{3}\pi\rho_1 R_1^3 c n_1. \tag{11}$$

Fig. 2. Distribution of the concentration of ice embryos $n_2(x, \tau)$, propagating in a crystallizing cloud mass (vertical section along the direction of propagation of crystallization)

The terms A and C do not play a decisive role in ensuring inequality (8), since the first becomes small as $u \to u_2$, and the second becomes equal to zero beginning from the moment of complete evaporation of the first droplets. As for the term B, the order of its magnitude may be estimated as

$$B \sim c(u_1 - u_2). \tag{12}$$

Proceeding from the considerations set forth, one can obtain the estimated inequality

$$D > B + E. \tag{13}$$

From inequality (13), by virtue of relations (9), (11), and (12), an approximate lower estimate is obtained for the value of the concentration of ice embryos at which the propagation of crystallization in a supercooled cloud mass is possible:

$$n_2 > \frac{w + \Delta u}{m_2}, \tag{14}$$

where $\mathcal{W} = \frac{4}{3}\pi\rho_1 R_1^3 n_1$ is the water content of the cloud, $\Delta u = u_1 - u_2$ is the difference between the vapor-concentration values corresponding to its equilibrium with a water and an ice surface at the cloud temperature, and $m_2 = \frac{4}{3}\pi\rho_2 R_2^3$ is the mass of a falling crystal.

Inequality (14) indicates that there exists a value of the concentration of crystals at the boundary of the frontal zone such that only beginning with it does the crystallized zone move toward the liquid phase. If the concentration of crystals n_2 is less than this value, then the liquid phase moves toward the crystalline one. We shall call this critical value of the concentration the **frontal concentration** and denote it by n_2^*.

Inequality (14) gives an approximate idea of the value of the frontal concentration. From the system of equations (1)–(7), as a result of solving it on an electronic computer, a more precise value of this concentration can be established for various values of the parameters $\mathcal{W}$, k_l, l, R_1, R_2, and the cloud temperature.

In the diffusive propagation of nuclei in a cloud mass, complete crystallization in one or another part of it does not occur until the frontal concentration of crystalline nuclei approaches this volume. Consequently, the motion of the crystallization front coincides with the motion of the frontal concentration of crystalline nuclei.

On the basis of the results obtained, the investigation of the propagation of crystallization in supercooled clouds, in a first approximation, can be reduced to the solution of a purely diffusion problem for the field of crystal concentration n_2. As was already said above, we regard the process of formation of the concentration n_2 as infinitely slow in comparison with the process in the zone, and the size of the zone as infinitely small in comparison with the size of the region in which the propagation of ice nuclei occurs (Fig. 2). As an additional condition for the diffusion problem it is necessary to take:

$$n_2 = 0 \quad \text{for } n_2 \leqslant n_2^*. \tag{15}$$

The motion of the frontal concentration, from a qualitative point of view, is analogous to the motion of a prescribed concentration of an impurity. In the case of an instantaneous impurity source acting at the initial instant of time, the prescribed impurity concentration first moves away from the source, then stops and moves in the opposite direction. The frontal concentration behaves in exactly the same way.

The qualitative character of the motion of the frontal concentration fully corresponds to the regularities in the behavior of crystallized zones in cloud masses. In experiments on the crystallization of supercooled clouds, at first an expansion of the crystallized regions is observed, then the expansion ceases and the crystallized zones begin to be drawn in by the liquid phase.

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Note: Figure translations are in progress. See original paper for figures.