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Abstract

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ON THE RADIATION OF A CHARGE MOVING IN A PLASMA NEAR CYCLOTRON RESONANCE

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As is well known, a charge moving in a magnetic field B in a circle with velocity $v_\perp \ll c$, in vacuum radiates mainly at the cyclotron frequency $\omega_B = eB/mc$. The radiation of a charge in a plasma located in a magnetic field differs from that in vacuum by a number of features associated both with the peculiarity of the polarization of waves with frequency $\omega = \omega_B$ ([1–4]), and with the anomalous dispersion of the plasma refractive index N for a wave of one of the two polarizations possible in a plasma as in an anisotropic medium. One of the features of this radiation, noted in the works of V. L. Ginzburg and V. V. Zheleznyakov ([1]) and others ([2–4]), is that in the dipole approximation the radiation of a charge in a plasma at the cyclotron frequency is absent. This result, however, was obtained for the case when the velocity of the charge $v_\parallel$ along the line of force of the magnetic field is strictly equal to zero. In fact, even a comparatively small velocity of longitudinal motion can lead, because of the Doppler effect, to a very substantial shift (owing to the large value of the refractive index near resonance) of the radiated frequency from the cyclotron frequency into a region where dipole radiation becomes possible. This radiation must evidently depend on the value of the refractive index N, and consequently on the plasma density n_0. Therefore the question arises whether this radiation can be used for measuring the plasma density. To answer this question, a numerical calculation was undertaken of the radiation intensity of an electron and an ion near the corresponding cyclotron frequencies. The velocity of the radiating electron (ion) is assumed to be considerably greater than the thermal velocities of the electrons (ions) of the plasma. In this case the plasma may be regarded as transparent for the radiation under consideration, and the refractive index of a "cold" plasma may be used in the calculations.

The radiation intensity of an electron is determined by the formula (first obtained by V. Ya. Eidman ([5]); see also ([4]))

$$I = \frac{e^2\omega_B^2}{c}\int_{-1}^{1} d\mu \left\{ \frac{x^2}{1+\alpha_x^2}\left[\left(\alpha_{x_0}\frac{\beta_1}{\lambda}+\alpha_{z_1}\beta_2\right)J_1(\lambda)-\beta_1 J_1'(\lambda)\right]^2 \frac{N}{|1-\beta_2\mu\,\partial(xN)/\partial x|}\right\}, \tag{1}$$

where

$$\lambda = \beta_1 x N\sqrt{1-\mu^2}, \qquad \beta_1 = \frac{v_\perp}{c}, \qquad \beta_2 = \frac{v_\parallel}{c},$$

$$x \equiv \frac{\omega}{\omega_B^0}\sqrt{1-\beta_1^2-\beta_2^2} = \frac{1}{1-\beta_2\mu N(x,\mu)},$$

$$N^2 = 1 - \frac{\frac{A^2}{x^2}(A^2-x^2)+x_0^2(1-\mu^2)\pm\sqrt{x_0^4(1-\mu^2)^2+4\frac{x_0^2}{x^2}(A^2-x^2)^2\mu^2}}{2\left[A^2+x_0^2-x^2-\frac{A^2x_0^2}{x^2}\mu^2\right]}.$$

$$\alpha_{x_0} = x\,\frac{N^2(x^2-x_0^2)-(x^2-x_0^2-A^2)}{A^2x_0}, \qquad \alpha_x = \alpha_{x_0}\mu - \alpha_{z_0}\sqrt{1-\mu^2},$$

$$\alpha_{z_0} = \frac{x^3N^2\{(x^2-x_0^2)N^2-(x^2-x_0^2-A^2)\}\mu\sqrt{1-\mu^2}}{A^2x_0\left[x^2N^2\sqrt{1-\mu^2}-(x^2-A^2)\right]},$$

$$x_0 = \frac{\omega_B^0}{\omega_B} = \frac{1}{\sqrt{1-\beta_1^2-\beta_2^2}}, \qquad A^2 = \frac{\omega_0^2}{\omega_B^2}, \qquad \omega_0^2 = \frac{4\pi e^2 n_0}{m_0}, \qquad \omega_B^0 = \frac{eB}{m_0 c}.$$

The integration in (1) is carried out over those values of μ for which $N^2 > 0$.

Table 1

Ratio of the radiation intensity of an electron in plasma to that in vacuum, $f_e = I/I_0$, and the relative optimal radiation frequency $x_{\text{opt}} = \omega_{\text{opt}}/\omega_B$, as functions of $\beta_1 = v_\perp/c$, $\beta_2 = v_\parallel/c$, $A^2 = \omega_{0e}^2/\omega_B^2$

	β_2	$\beta_1 = 0.01$, $A^2 = 3$	$\beta_1 = 0.01$, $A^2 = 10$	$\beta_1 = 0.01$, $A^2 = 100$	$\beta_1 = 0.5$, $A^2 = 3$	$\beta_1 = 0.5$, $A^2 = 10$	$\beta_1 = 0.5$, $A^2 = 100$	$\beta_1 = 0.9$, $A^2 = 3$	$\beta_1 = 0.9$, $A^2 = 10$	$\beta_1 = 0.9$, $A^2 = 100$
$x_{\text{opt}} f_e$	0.01	0.890.64	0.812.2	0.6011	0.940.36	0.910.32	0.820.18	0.980.54	0.970.38	0.920.24

	β_2	$\beta_1 = 0.01$, $A^2 = 3$	$\beta_1 = 0.01$, $A^2 = 10$	$\beta_1 = 0.01$, $A^2 = 100$	$\beta_1 = 0.5$, $A^2 = 3$	$\beta_1 = 0.5$, $A^2 = 10$	$\beta_1 = 0.5$, $A^2 = 100$	$\beta_1 = 0.9$, $A^2 = 3$	$\beta_1 = 0.9$, $A^2 = 10$	$\beta_1 = 0.9$, $A^2 = 100$
$x_{\mathrm{opt}} f_e$	0.1	0.182.4	0.156.3	0.095.8	0.700.44	0.630.31	0.350.09	0.840.47	0.770.29	0.510.11
$x_{\mathrm{opt}} f_e$	0.7	0.091.6	0.071.3	0.040.64	0.330.35	0.260.20	0.100.14			

The calculation of the radiation intensity by these formulas was carried out by one of the authors (A. E. Bazhanova) on the high-speed M-20 computer. The calculation showed that the main part of the radiation falls within a comparatively narrow frequency range near a certain optimal frequency $\omega = \omega_{\mathrm{opt}}$.

Table 2

Dependence of $x_{\mathrm{opt}} = \omega_{\mathrm{opt}}/\omega_{B_i}$ and of the coefficient f_i, which determines the relative radiation intensity of an ion,

$$\frac{I}{I_0} = \frac{c}{c_A} f_i(\beta_1, \beta_2) \quad \text{on} \quad \beta_1 = v_\perp / c_A, \quad \beta_2 = v_\parallel / c_A$$

β_2	$\beta_1 = 0.1$	$\beta_1 = 0.5$	$\beta_1 = 1$	$\beta_1 = 3$	x_{opt}
0.1	22	0.98	0.22	0.037	0.82
0.5	25	1.2	0.30	0.029	0.60
1	21	1.1	0.27	0.028	0.45
3	12	0.64	0.17	0.020	0.24

The frequency ω_{opt} depends rather strongly on the plasma density. Table 1 gives the dependence of $\omega_{\mathrm{opt}}/\omega_B$ on A^2 (i.e., in fact, on the plasma density) for several values of β_1 and β_2. Since for $\omega > \omega_B^0$, $N^2 < 0$, it is clear in advance that the charge can radiate only for $\mu < 0$ (μ is the cosine of the angle between the projection of the velocity vector onto the field line and the normal to the front of the emitted wave) and $\omega < \omega_B^0$. The corresponding frequency range is known as the propagation region of "whistling atmospherics" ([6]). The group velocity of the waves in this range makes a comparatively small angle with the vector of the magnetic field. It follows from what has been said that a charge moving along the magnetic field will radiate mainly backward. The radiation intensity proves to be comparable with the intensity of dipole radiation in vacuum,

$$I_0 = \frac{2}{3} \frac{e^2}{c^3} \omega_B^2 v_\perp^2.$$

The ratio of these intensities, $f_e = I/I_0$, is given in Table 1.

The data presented allow one to hope for the possibility of using the radiation of fast electrons (specially passed through plasma or present in it, as, for example, “runaway electrons” in powerful discharges) in order to obtain certain information about the plasma.

An analogous calculation was carried out for ions. Ion radiation may be of interest in the analysis of cosmic radio emission. In Ref. ([3]) a rough estimate was given for the radiation of an ion having a longitudinal velocity $v_\parallel \neq 0$ near cyclotron resonance. This estimate shows that the radiation intensity of an ion, even for comparatively small $v_\parallel$, is not only nonzero (as it would be for $v_\parallel = 0$), but also considerably exceeds the intensity of the dipole radiation of an ion in vacuum. This increase in the radiation intensity, associated with the large value of the refractive index N (in the frequency region under consideration $N \sim c/c_A$, where $c_A = \sqrt{B^2/4\pi M n_0}$ is the “Alfvén velocity” ; values $N \sim 10^4$ are quite realistic), can make ion radiation a fully observable effect.

The intensity of ion radiation near cyclotron resonance is determined by the formula ([4,5]) ($c_A \ll c$)

$$I = \frac{e^2\omega_{Bi}^2}{cA}\int_{-1}^{1} d\mu \left\{ \frac{x^2}{1+\alpha_x^2}\left[\left(\alpha_{x_0}\frac{\beta_1}{\lambda} + \alpha_{z_0}\beta_2\right) J_1(\lambda) + \beta_1 J_1'(\lambda)\right]^2 \frac{n}{|1-\beta_2\mu\,\partial(xn)/\partial x|}\right\}, \tag{2}$$

where

$$\lambda = \beta_1 x n\sqrt{1-\mu^2}, \qquad \beta_1 = \frac{v_\perp}{c_A}, \qquad \beta_2 = \frac{v_\parallel}{c_A},$$

$$x = \frac{\omega}{\omega_{Bi}} = \frac{1}{1-\beta_2 n\mu}, \qquad \alpha_x = -\frac{\mu x}{n^2\{\xi x^2(1-\mu^2) - (1-x^2)\mu^2\} + 1},$$

$$\alpha_{x_0} = \frac{n^2(1-x^2) - 1}{x}, \qquad \alpha_{z_0} = -\frac{\xi n^2 x^3 \mu\sqrt{1-\mu^2}}{n^2\{\xi x^2(1-\mu^2) - (1-x^2)\mu^2\} + 1},$$

$$n^2 = \frac{1+\mu^2+\sqrt{(1-\mu^2)^2 + 4x^2\mu^2}}{2\{\mu^2 - x^2[\mu^2 + \xi(1-\mu^2)]\}},$$

$$c_A = \sqrt{\frac{B^2}{4\pi M n_0}}, \qquad \xi = \frac{m}{M} = \frac{1}{1840}, \qquad A^2 = \frac{\omega_{0i}^2}{\omega_{Bi}^2} = \frac{c^2}{c_A^2}.$$

The integration in (2) is performed over those values of μ for which $n^2 > 0$.

The calculation shows that the radiation occurs practically at one frequency ω_{opt}, which can be shifted rather strongly relative to the resonance (see Table

2). The ratio of the intensity of ion radiation in plasma to that in vacuum may be written in the form

$$\frac{I}{I_0} = Af_i(\beta_1, \beta_2).$$

The values of f_i are given in Table 2. As can be seen, these values are of order unity, so that the radiation intensity, roughly speaking, is A times greater than the vacuum value. We note that in an isotropic medium with refractive index $N = A$ one would have $f_i = 1$.

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Note: Figure translations are in progress. See original paper for figures.