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Abstract

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Complex Steady-State Motions in Nonlinear Impulse Systems

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1. In nonlinear automatic systems, alongside simple periodic regimes, steady-state motions of a more complex nature may arise. Among these motions there exist some that admit approximation by some single predominant harmonic. To determine complex steady-state motions of this kind, an approximate frequency method can be developed, analogous to the harmonic-balance method (^{1}).

2. Consider a nonlinear impulse system (^{2}), whose simple periodic regimes were studied in (^{3}). We shall retain, wherever possible, the notation adopted in (^{3}).

Let a motion have been established in this system which, at the input of the nonlinear (impulse) element, can be approximated by the predominant harmonic

$$u(\bar{t}) = C\cos\left(\frac{2\pi}{N}\bar{t} + \psi\right)$$

or, in complex form,

$$U = Ge^{i\psi}. \tag{1}$$

The quantity N—the relative period—is, in the general case, any positive real number.

Let us determine this motion approximately.

3. Suppose first that $N > 0$ is rational, and denote

$$N = N_0/N_1 = M_0/M_1,$$

where N_0, N_1 are relatively prime numbers, and

$$M_i = N_i[1 + N_0 - 2E(N_0/2)].$$

Fig. 1: spectra labeled amplitude vs frequency, panels a and b

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Similarly to the harmonic-balance method, we shall characterize the nonlinear element by an equivalent complex gain coefficient, which we define by the expression

$$J^*_{M_1} = \frac{Z_{M_1}}{U}. \tag{2}$$

Here Z_{M_1} is the complex amplitude of the M_1-th, predominant harmonic of the periodically varying output quantity of the nonlinear element, to whose input the sinusoidal action (1) is applied.

The approximate condition for the existence of the desired steady-state motion, similarly to how this is written in (^{3}), can be represented in the form

$$W\left(j\frac{2\pi}{N}\right) = -\frac{1}{J^*_{M_1}}. \tag{3}$$

4. The expression for the M_1-th harmonic at the output of the nonlinear impulse element for amplitude, width, and time impulse modulations and an arbitrary nonlinearity in the impulse element can be represented, using the approach of (^{3}), in the form

$$Z_{M_1} = \sum_{s=0}^{M_0-1} A_s, \tag{4}$$

where

$$A_s = a_s K_n \varphi_{Y_s}\left(j\frac{2\pi}{N}\right) a_{M_1}(s+\beta_s), \qquad a_{M_1}(s) = \frac{2}{M_0 T_0} e^{-j\frac{2\pi}{N}s}.$$

For amplitude-pulse modulation, as in (3), one may write

$$Z_{M_1} = \sum_{l=-\infty}^{\infty} B_{1+lN}, \qquad l = \lambda M_1. \tag{5}$$

If $N_1 = 1$, then $N = N_0$, and relations (4), (5) reduce to the expressions for the first harmonic used in determining simple periodic regimes by the harmonic-balance method (3).

Fig. 1

Relations analogous to (5) for width and time modulation represent expansions of the predominant harmonic in series in Bessel functions of the first kind.

5. Let now $N > 0$ be irrational. Consider the sequence $\{N(n)\}$ of approximations to the quantity N accurate to n digits. To each member $N(n)$ of this sequence there corresponds some predominant harmonic $Z_{M_1(n)}$. The limit of the sequence $\{Z_{M_1(n)}\}$ determines the predominant harmonic for the given N.

From the limiting relation

$$N = \lim_{n\to\infty} N_0(n) = \lim_{n\to\infty} \frac{N_0(n)}{N_1(n)}$$

it follows that

$$\lim_{n\to\infty} N_0(n) = \lim_{n\to\infty} N_1(n) = \infty,$$

whence, for amplitude-pulse modulation, using (5), we obtain directly

$$Z_\infty = \lim_{n\to\infty} Z_{M_1(n)} = B_1. \tag{6}$$

In other words, for irrational N in the case of amplitude-pulse modulation, the predominant harmonic, to within a factor, coincides with the first harmonic of the response to (1) of an ordinary nonlinear element.

In an analogous way, from the corresponding series, expressions for the predominant harmonic Z_∞ can be obtained for width and time modulations.

The equivalent complex gain coefficient of the nonlinear pulse element and the approximate condition for the existence of the motion are determined here similarly to (2) and (3). This makes it possible to adopt expressions (2) and (3) as a unified form for writing the indicated relations for any N.

The graphical solution of equation (3) consists in finding the mutual intersection of the frequency characteristic $W(j\omega)$, plotted in modulus-phase-frequency coordinates, with the right-hand side of equation (3) plotted in the same coordinates.

6. The exposition above presupposes the admissibility of approximating the steady-state motion by one, predominant harmonic. As is the case in the approximate determination of simple periodic regimes, establishing rigorous limits for the justified use of the developed method constitutes a special and rather difficult problem and is not considered here.

As examples of steady-state motions admitting approximation by a predominant harmonic, one may cite complex periodic regimes in relay amplitude-pulse systems with a hold device ($K_n = 6.5$; $\gamma = 1$). For the case $W(pT_0) = 1/p\,(p^2 + 0.1p + 1)$, $p = j\omega$, $T_0 = 1.37$ sec., $a_n = \operatorname{sign} U[n]$, the 11th harmonic predominates (Fig. 1a), $N = 4.73$, $c = 72$; for the case $W(pT_0) = 2.3/p\,(p + 1)$, $T_0 = 1.045$ sec.,

$$a_n = {}^1/_2 \sum_{i=-1}^{2} \operatorname{sign}\left(u[n] + i - {}^1/_2\right)$$

the second harmonic predominates (Fig. 1b), $N = 4.5$, $c = 14.6$.

An example of a steady-state motion admitting approximation by a predominant harmonic is also furnished by the process given in [4] (p. 290, Fig. 9) and interpreted there as a phenomenon of random nature. From the standpoint set forth here, the indicated process is a complex steady-state motion in which the harmonic with frequency 0.208 Hz predominates.

7. The problem of the approximate determination of complex steady-state motions in systems with an ordinary, non-pulse nonlinearity is a special case (as $T_0 \to 0$) of the problem considered above and is solved in an analogous manner.

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Note: Figure translations are in progress. See original paper for figures.

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