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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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CYBERNETICS AND CONTROL THEORY

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STABILITY OF PROCESSES IN NONLINEAR PULSE AUTOMATIC SYSTEMS

(Presented by Academician B. N. Petrov on March 28, 1963)

Until recently, the theory of stability of nonlinear pulse automatic systems (NPAS) was limited to establishing sufficient conditions for stability of the equilibrium position. For investigating the stability of the equilibrium position of NPAS "as a whole," analogues of A. M. Lyapunov's direct method⁽¹⁻⁷⁾ and of V. M. Popov's method⁽⁸⁻¹⁰⁾ were developed. However, in many cases, when an NPAS is intended to reproduce or transform an input action, the problem arises of investigating established or forced processes in an NPAS.

Fig. 1

In the present work it is shown that results obtained earlier in the study of stability in the large of the equilibrium position⁽⁸⁾ can be used to investigate the stability of processes in NPAS, and conditions are found which ensure stability of processes under arbitrary deterministic or random input actions.

Consider an NPAS consisting of a nonlinear element (NE) and a linear pulse part (LP) (Fig. 1a). The NE has the characteristic $\Phi(x)$, belonging to the sector $(0, k)$ (Fig. 2), i.e., satisfying the conditions⁽⁸⁾

$$\text{a) } \Phi(0) = 0; \quad \text{b) } 0 \leq \frac{\Phi(x)}{x} < k; \quad \text{c) } \lim_{x \rightarrow \infty} \Phi(x) \neq 0. \quad (1)$$

The LP includes a pulse element and the reduced continuous part. Let $w[n]$, $W^*(q)$, $W^*(j\omega)$ denote respectively the impulse characteristic, the transfer function, and the frequency characteristic of the LP⁽¹¹⁾. For a stable LP the poles of $W^*(q)$ have negative real parts, and $w[n] \rightarrow 0$ as $n \rightarrow \infty$. We shall henceforth consider this case.

Fig. 2

Figure 2: Fig. 2

Suppose that in the NPAS there is a steady-state (or forced) process $z^0[n]$, caused by the input action $f^0[n]$. The error in the steady-state process is equal to

$$x^0[n] = f^0[n] - z^0[n]. \quad (2)$$

Assume that a vanishing disturbing action is applied to the NPAS, i.e., an action tending to zero as $n \rightarrow \infty$. This disturbance-

the perturbing action $f_n[n]$ may be caused by the initial conditions and by an instantaneous disturbance of the forcing action. In this case the error will change and become equal to

$$x[n] = x^0[n] + \xi[n], \quad (3)$$

where $\xi[n]$ is the deviation.

We shall say that the established process in an NIPACS is stable as a whole if, for any vanishing perturbing action, the deviation $\xi[n]$ satisfies the condition

$$\lim_{n \rightarrow \infty} \xi[n] = 0. \quad (4)$$

It can be shown that this definition corresponds to Lyapunov's definition of asymptotic stability "in the large." Introducing the perturbing action at the input of the NE, we write the equation of the NIPACS in the form ^(8,10)

$$x[n] = f^0[n] + f_n[n] + \sum_{m=0}^n w[n-m]\Phi(x[m]). \quad (5)$$

Fig. 2

For the investigated established (or forced) process, with $f_n[n] \equiv 0$, we obtain

$$x^0[n] = f^0[n] - \sum_{m=0}^n w[n-m]\Phi(x^0[m]). \quad (6)$$

Subtracting (6) from (5), we find the equation with respect to $\xi[n]$

$$\xi[n] = f_n[n] - \sum_{m=0}^n w[n-m]\Phi_1(\xi[m], m), \quad (7)$$

Fig. 3

Figure 3: Fig. 3

where

$$\Phi_1(\xi[n], n) = \Phi(x^0[n] + \xi[n]) - \Phi(x^0[n]). \quad (8)$$

This equation corresponds to a nonstationary NIPACS, whose structural diagram is obtained from the structural diagram of the stationary NIPACS (Fig. 1a) by replacing the NE with characteristic $\Phi(x)$ by an NE with characteristic $\Phi_1(\xi, n)$, which is also a function of time (Fig. 1b). The study of stability “as a whole” of the established process is reduced to the study of stability of the equilibrium position of the nonstationary NIPACS.

Fig. 3

The characteristic of the NE of the nonstationary NIPACS always satisfies condition (1a). Suppose that, for $n \geq n_0$, it also satisfies conditions (1b) and (1c). Then, repeating literally the derivation of the stability criterion that was given in (8), with replacement of the auxiliary function

$$\varphi_N[n] = \begin{cases} \Phi(x[n]), & 0 \leq n \leq N, \\ 0, & n < 0, n > N, \end{cases}$$

by

$$\varphi_N[n] = \begin{cases} \Phi_1(\xi[n], n), & 0 \leq n \leq N, \\ 0, & n < 0, n > N, \end{cases}$$

we arrive at the following criterion for stability of the process “as a whole.”

In order that the process in an NIPACS be stable “as a whole,” it is sufficient that the NE characteristic $\Phi_1(\xi[n], n)$, beginning with $n > n_0$, belong to the sector $(0, k)$, where k is determined from the condition

$$\operatorname{Re} W^*(j\bar{\omega}) + \frac{1}{k} > 0. \quad (9)$$

The quantity k is determined by the abscissa of the vertical tangent to the frequency characteristic of the LE (Fig. 3).

If the process $x^0[n]$ is known, then from the characteristic $\Phi(x) = \Phi(\xi)$ it is easy to construct the characteristic $\Phi_1(\xi, n)$ for each n . To do this, one must shift $\Phi(\xi)$ along the abscissa axis by $-x^0[n]$ and along the ordinate axis by the constant quantity $\Phi(x^0, [n])$, or, in other words, move the origin of coordinates

Fig. 4

Figure 4: Fig. 4

along the characteristic to the point corresponding to $\xi = -x^0[n]$ (Fig. 4). Applying the stability criterion formulated above, we conclude that if the family of characteristics obtained in this way belongs, for all $n > n_0$, to the sector $(0, k)$, then the process under study in the nonlinear pulse automatic system is stable.

Fig. 4

Application of this criterion in each specific case presents no difficulty if $x^0[n]$ is known. But determining $x^0[n]$, as a rule, is extremely difficult. It is therefore important to determine conditions that ensure stability of the process as a whole under any input action.

On the basis of the method of constructing $\Phi_1(\xi, n)$ from $\Phi(x)$, it is easy to conclude that if $\Phi(x)$ belongs to the sector $(0, k)$ and $d\Phi(x)/dx$ satisfies the inequality

$$0 \leq \frac{d\Phi(x)}{dx} < k, \quad (10)$$

then $\Phi_1(\xi, n)$ will also belong to the sector $(0, k)$ for any n .

Thus, the following is true.

Theorem. A nonlinear pulse automatic system with an NE characteristic that is a nondecreasing function with maximum steepness not exceeding k is stable “as a whole” under any input action. For such nonlinear pulse automatic systems, stability of the equilibrium position guarantees stability of the process under any prescribed external action.

This theorem also extends to the case when the prescribed actions are random, or when constantly acting disturbances caused by variations of the equations of the nonlinear pulse automatic system are applied to the nonlinear pulse automatic system.

Above, the case of a stable LE was considered. It is not difficult to extend the theorems to the case of a neutral* and unstable LE, if condition (10) is replaced by the condition

$$r < \frac{d\Phi(x)}{dx} < k + r, \quad (11)$$

where for a neutral LE r is an arbitrarily small quantity, and for an unstable LE r is the value of the gain coefficient of the linear element which, when inserted in place of the nonlinear element, stabilizes the system linearized in this way (9); here k is determined from condition (9)

$$\operatorname{Re} \frac{W^*(j\omega)}{1 + rW^*(j\omega)} + \frac{1}{k} > 0. \quad (12)$$

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* For a neutral LE, $W^*(y)$ has one zero pole.

Note: Figure translations are in progress. See original paper for figures.

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