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1.** Consider a certain polynomial

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Abstract

Full Text

CYBERNETICS AND CONTROL THEORY

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ON THE MINIMIZATION OF THE TEMPORAL MEASURE OF A MULTILAYER CYCLE

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1. Consider a certain polynomial

$$\varphi = \sum_{j=0}^{n} b_{i_j} \prod_{l=0}^{j} a_{i_l},$$

where $p_{i_j} = (a_{i_j}, b_{i_j})$ are prescribed pairs of numbers, $a > 1$, $b > 0$, and $i_j \in \{1, 2, \dots, n\}$; $j = 1, 2, \dots, n$; $i_{j_1} \neq i_{j_2} \leftrightarrow j_1 \neq j_2$. It is required to find such an arrangement of the pairs in φ that $\varphi_0 = \min\{\varphi\}$.

Theorem 1. The substitution

$$\begin{pmatrix} 0, & 1, & \dots, & n \\ i_0, & i_1, & \dots, & i_n \end{pmatrix},$$

defined by the chain of inequalities

$$b_{i_n} + \frac{b_{i_n}}{a_{i_n} - 1} \leq b_{i_{n-1}} + \frac{b_{i_{n-1}}}{a_{i_{n-1}} - 1} \leq \dots \leq b_{i_0} + \frac{b_{i_0}}{a_{i_0} - 1}, \tag{1}$$

gives $\min \varphi\,(p_{i_0}, p_{i_1}, \dots, p_{i_n})$.

Lemma. If a transposition in $(i_0, i_1, \dots, i_n)$ causes an inversion in (1), then the corresponding value of φ is not less than the preceding one.

Theorem 1 follows from the lemma.

2. For what follows we give some definitions.

By a **cycle** we shall mean a closed sequence of commands (or operators) ([2]). The predetermined number of realizations of a cycle will be called its **multiplicity** ([2]).

If C_1 and C_2 are two cycles, with $C_2 \subset C_1$, then the cycle C_1 will be called the **envelope** of C_2 ([1]), and $C_1 - C_2 = C_1 - C_1 \cap C_2$ the **envelope of the cycle** C_2

with respect to C_1 ([1]). Otherwise, C_2 is a **two-layer cycle**. If a sequence of cycles $C_0, C_1, \dots, C_n$ is given, with $C_n \subset C_{n-1} \subset \dots \subset C_1 \subset C_0 = C$, then each cycle C_{j-1} will be called the **envelope** of C_j; $C_{j-1} - C_j = C_{j-1} - C_j \cap C_{j-1}$ is the $(j-1)$-st **envelope of the cycle** C, and, correspondingly, the cycle C is an $(n+1)$-layer cycle. Otherwise, C is a **multilayer cycle**.

Let some multilayer cycle C process a partially commutative direct product of tables $F = f_1 \times f_2 \times \dots \times f_m$, with the number of components of the tables equal to the multiplicities of the corresponding closures.

The factors in F are ordered, guided by the functional subordination of the parameters. In this case groups are distinguished in F within which the factors are commutative.

We shall be interested in finding such a structure in F that, all other conditions being equal, the realization time of the corresponding cycle C is minimal.

It is clear that under a permutation of tables in F, the corresponding layers in C (we shall call them **commutative**) likewise, being modified, are moved.

Denote by t_{i_j} the time required for a single realization of the envelope of the i_j-th cycle, and by $T(C)$ the time of a complete realization of the cycle C, i.e., its temporal measure.

3. **Idealized model 1.** Suppose that the t_{ij} corresponding to commutative layers are invariant with respect to the arrangement of the layers in C. Then the following holds.

Theorem 2. *If the t of the commutative layers of the multilayer cycle $C(F)$ are invariant with respect to the arrangement of the layers in C, then, all other conditions being equal, minimization of $T(C)$ is ensured by ordering the commutative layers in the commutativity groups F according to (1), where $a_{ij} = k(\lfloor\_i\rfloor)$ is the multiplicity of the cycles of the corresponding commutative layers of group i; $b_{ij} = f_{ij}$.*

Otherwise, (1) can be rewritten in the form

$$t_{i_n} + \frac{t_{i_n}}{k(\lfloor\_i_n\rfloor) - 1} \le t_{i_{n-1}} + \frac{t_{i_{n-1}}}{k(\lfloor\_i_{n-1}\rfloor) - 1} \le \dots \le t_{i_0} + \frac{t_{i_0}}{k(\lfloor\_i_0\rfloor) - 1}. \quad (1^*)$$

The proof follows from Theorem 1.

4. **Idealized model 2.** Let $t_k = \delta t_{ij} + \Delta_k$, where $\delta t_{ij} > 0$, $\Delta_k \ge 0$ are fixed quantities, with Δ_k invariant with respect to the ordinal number of the layer $k = i_j$, and δt_{ij} invariant for the fixed layer i.

Theorem 3. *If, for (1), $a_{i_n} \le a_{i_{n_1}} \le \dots \le a_{i_0}$, then, all other conditions being equal, (1) determines $\min T(C)$, where $b_{ij} = \delta t_{ij}$, $a_{ij} = k(\lfloor\_i\rfloor)$.*

5. **Remark.** In order for Theorems 2 and 3 to be applicable in constructing real multilayer cycles, it is necessary that either $T(O_{ij}) \leq t_{i_{j-1}}$, or $T(O_{ij}) \simeq T(O_{i_{j-1}})$.

6. If it is assumed that the changes in t_{ij} under permutation of layers are so small that relation (1) is not violated, then one can see that the limits of applicability of Theorems 2 and 3 are widened.

7. Thus, Theorems 2 and 3 indicate, in the models under consideration, in what order the commutative layers of the multilayer cycle C should be arranged so that the time measure $T(C)$ is minimal.

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CITED LITERATURE

1. Yu. I. Yanov, in: *Problems of Cybernetics*, 1, Moscow, 1958.

2. V. A. Zatitskii, *Proceedings of the Seminar "Modern Digital Automation and Computer Technology"*, 2, Moscow, 1962.

Note: Figure translations are in progress. See original paper for figures.