Yu. P. Raiser

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Abstract

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Yu. P. Raiser

MOTION IN AN INHOMOGENEOUS ATMOSPHERE CAUSED BY A SHORT-DURATION PLANE IMPACT

(Presented by Academician Ya. B. Zel̆ dovich, 17 VI 1963)

HYDROMECHANICS

1. In the works of A. S. Kompaneets (^{1,2}), a strong point explosion in an inhomogeneous atmosphere is considered approximately. The solution (^{1,2}) loses validity when the shock wave moves upward from the point of explosion to very large distances and the pressure in the cavity that has formed falls to a very small value. The further propagation of the shock wave downward has much in common with the motion considered in the well-known problem of a short-duration impact on the surface of a gas bordering on a vacuum ($^{3-5}$).

Below the problem of a short-duration impact is solved for the case of an inhomogeneous atmosphere.

2. Let the initial density be distributed in space according to the barometric law

$$\rho_0 = \rho^* e^{x/\Delta}, \qquad \Delta = \text{const.} \tag{1}$$

The initial pressure is equal to zero. At the moment $t = 0$, in a region of very small density at $x \approx -\infty$, a plane impact or explosion is produced. A shock wave runs through the gas in the direction of increasing density, while the heated gas expands toward the vacuum. In the limiting motion there is a length scale Δ, but no scales of density or time; the coordinate x is determined only up to an additive constant*. Therefore the velocity of the shock-wave front is

$$D = \dot{X} = \alpha \frac{\Delta}{t}, \tag{2}$$

where the coefficient α depends only on the adiabatic exponent γ. The coordinate of the front increases with time as

$$X = \alpha \Delta \ln t + \text{const.} \tag{3}$$

The shock wave captures a mass of gas

$$M = \int_{-\infty}^{X} \rho_0 \, dx = \rho_0(X)\Delta, \tag{4}$$

which, by virtue of the condition $\dot{M} = \rho_0(X)\dot{X}$, is equal to

$$M = At^{\alpha}. \tag{5}$$

The integration constant A, of dimension $\mathrm{g} \cdot \mathrm{cm}^{-2} \cdot \mathrm{s}^{-\alpha}$, is a parameter characterizing the intensity of the impact.

3. The dimensional features of the problem make it possible to represent the velocity, density, and pressure of the gas in the form

$$u = \frac{2}{\gamma+1}\alpha\frac{\Delta}{t}\, v = u_\phi v, \qquad \rho = \frac{\gamma+1}{\gamma-1}\frac{At^{\alpha}}{\Delta}\, q = \rho_\phi q, \tag{6}$$

$$p = \frac{2}{\gamma+1}\alpha^2\frac{\Delta A}{t^{2-\alpha}}\, f = p_\phi f,$$

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[*] The quantity ρ^* is indeterminate because of arbitrariness in the choice of the origin for x.

where $u_\phi, \rho_\phi, p_\phi$ are the values at the shock-wave front, while the functions v, q, f depend only on the dimensionless distance $\xi \equiv \dfrac{X-x}{\Delta}$, measured from the front, and on γ. At the shock-wave front, for $\xi = 0$, $v = q = f = 1$.

The motion has a somewhat unusual self-similarity: the profiles of the gas-dynamic quantities move together with the shock-wave front, without stretching with time. However, in Lagrangian coordinates the motion is self-similar in the usual sense. Indeed, the mass Lagrangian coordinate is equal to

$$m = \int_{-\infty}^{x} \rho(x)\, dx = \mathrm{const} \cdot M \int_{\xi}^{\infty} q(\xi)\, d\xi,$$

i.e., ξ, and consequently v, q, f, are functions of the self-similar variable $\eta = m/M = m/At^{\alpha}$.

4. Substituting expressions (6) into the equations of gas dynamics written in Lagrangian coordinates,

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial m} = 0, \qquad \frac{\partial (1/\rho)}{\partial t} - \frac{\partial u}{\partial m} = 0, \qquad p\rho^{-\gamma} = F(m). \tag{7}$$

we obtain equations for the functions—the representatives $v(\eta), q(\eta), f(\eta)$:

$$v + \alpha\eta v' = \alpha f'; \qquad \frac{1}{q} + \eta\left(\frac{1}{q}\right)' = -\frac{2}{\gamma - 1} v'; \qquad f q^{-\gamma}\eta^{2/\alpha + \gamma - 1} = 1. \tag{8}$$

Integrating the second equation and eliminating q and v from the system, we find the basic equation of the problem:

$$\frac{df}{d\eta} = \frac{\gamma+1}{2\alpha}\,\frac{1 - \dfrac{\gamma-1}{\gamma+1}\left(1 - \dfrac{2-\alpha}{\gamma}\right) f^{-1/\gamma}\eta^{-(2-\alpha)/\alpha\gamma}}{1 - \dfrac{\gamma-1}{2\gamma} f^{-1/\gamma-1}\eta^{-(2-\alpha)/\alpha\gamma}}. \tag{9}$$

Since at $x = -\infty$, or $\eta = 0$, the pressure is zero, the solution must pass through two points

$$\eta = 1, \quad f = 1; \qquad \eta = 0, \ f = 0, \tag{10}$$

which determines the exponent α.

5. In the case $\gamma = 2$, it is possible to find an exact analytic solution of the problem*. We have: $\alpha = 3/2$, $M \sim t^{3/2}$, $\dot{X} = 3/2\,\Delta/t$,

$$u_\phi \sim 1/t, \quad \rho_\phi \sim t^{3/2}, \quad p_\phi \sim 1/t^{1/2},$$

$$f = \eta, \qquad q = \eta^{5/3}, \qquad v = {}^3/_2\left(1 - {}^1/_3\eta^{-2/3}\right). \tag{11}$$

In Eulerian coordinates:

$$f = (1 + 2\xi)^{-3/2}, \quad q = (1 + 2\xi)^{-5/2}, \quad v = 1 - \xi. \tag{12}$$

An analytic solution is also possible for $\gamma = 1$: $\alpha = 1$, $f = \eta$, $q = \eta^3$, $v = 1$. This case is of interest only from the point of view of bounding the exponent α, since it corresponds to infinite compression of the gas at the wave front, as a result of which, in Eulerian coordinates, v, q, f become δ-functions.

Since the real values of γ for gases lie in the range $1 < \gamma < 2$, it must be assumed that the corresponding values of α lie in the interval $1 < \alpha < 3/2$**.

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

* This solution, written in Lagrangian coordinates, is completely analogous to the analytic solution of the usual problem of a short-time impulse, found for the special case $\gamma = 7.5$ (5).

** Consideration of the energy and momentum balances, analogous to (3), leads to the general restriction $1 < \alpha < 2$.

Equation (9), as usual, has a singular point of saddle type, through which passes the integral curve corresponding to the solution. In the general case of an arbitrary γ, the exponent α and the solution are found by numerical integration by the trial method (see (4)). Figures 1 and 2 show the distributions, obtained in this way, of p, ρ, and u behind the shock wave for the case $\gamma = 1.25$. In this case $\alpha = 1.345$.

Fig. 1. Distribution of the pressure f, density q, and velocity v behind the shock wave in the mass coordinate

Fig. 2. Distribution of the pressure f, density q, and velocity v in space behind the shock wave

6. Let us return to the question of a point explosion in an inhomogeneous atmosphere. The motion of the shock wave downward acquires the character described here when the pressure in the cavity p_c becomes much smaller than the pressure at the shock front p_ϕ.* Suppose, for definiteness, that the transition to the new regime occurs at $p_\phi/p_c = 10$. According to (2), this value corresponds to the time from the moment of explosion $t_1 \approx 19\tau$, where the time scale τ is equal to $\tau = (\rho_{00}\Delta^5/E)^{1/2}$ (ρ_{00} is the air density at the point of explosion, E is the explosion energy). By the time t_1, the shock wave has moved downward from the point of explosion by a distance $z = 1.9\Delta$; the front velocity at this time is equal to $D_1 = 2.5 \cdot 10^{-2}\Delta/\tau$.

Let us extrapolate the limiting laws of propagation of the shock wave (3), (2) to the transition (“initial”) moment, and let us choose the coordinate and time in such a way that the initial condition $D = D_1$ at $X = 0$ is satisfied. We obtain, approximately with logarithmic accuracy,

$$X = \alpha\Delta \ln \frac{D}{D_1} = \alpha\Delta \ln \frac{t}{\theta},$$

where the parameters D_1 and θ are determined in terms of the explosion parameters by the expressions

$$D_1 = 2.5 \cdot 10^{-2}\Delta/\tau = 2.5 \cdot 10^{-2}(E/\rho_{00}\Delta^3)^{1/2}, \qquad \theta = 40\alpha\tau^{**}.$$

In the same approximation,

$$A = e^{1.9}\rho_{00}\Delta\theta^{-\alpha}.$$

An estimate using real numerical values of the parameters shows that, in the process of deceleration of the shock wave from the "transition" velocity D_1 to a velocity of the order of 1 km/sec, several times greater than the speed of sound in cold air, the shock wave travels downward a distance of order (2—3)Δ, which is added to the distance $\simeq 2\Delta$ that follows from the theory (^{1,2}).

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Note: Figure translations are in progress. See original paper for figures.

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* In a strong point explosion in a homogeneous atmosphere, $p_\phi/p_c \approx 2.5$ (6).

** Let us note that the numerical values of D_1 and θ depend only weakly on the choice of the transition value p_ϕ/p_c. Thus, at the last moment calculated in (2), $t \approx 23.4\tau$, close to the moment of "breakthrough" of the atmosphere, $z = 2.0\Delta$, $D_1 = 2.12 \cdot 10^{-2}\Delta/\tau$, $p_\phi/p_c = 22$.