CYBERNETICS AND CONTROL THEORY

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Abstract

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PROBLEMS OF OPTIMAL CONTROL OF PROCESSES OF DRAWING PRODUCTS FROM A MELT

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The problem of controlling the processes of drawing products from a melt in a number of cases consists in the exact observance of a prescribed size of the product being drawn. In other cases the requirements are imposed primarily on the structure of the product material ($^{1-7}$). As is shown below, these problems can be reduced to problems of optimal control of systems with distributed parameters, considered in ($^{13-15}$). For this it is necessary to give a description of the processes of drawing products from a melt.

The equation of the surface of the liquid column connecting the cylindrical body being drawn with the mass of melt in the crucible has the form

$$\frac{a^2}{\sqrt{F'^2+1}} = \frac{\nu}{F} - F(h+\mu) - F^3h^2b^2, \tag{1}$$

where $a^2 = 2\alpha/\rho g$, $b^2 = \pi^2 n_3^2/gh^{*2}$, ν and μ are arbitrary constants. Here α is the surface-tension coefficient, ρ is the density, g is the acceleration of free fall, n_3 is the rotation speed of the seed, h^* is the coordinate of the wetting perimeter, $F(h)$ is the solution, and h is the independent variable.

Since equation (1) is obtained by minimizing the free energy at fixed volume

$$\int_0^{h^*} F^2\,dh = \text{const},$$

for uniqueness of the solution it is necessary, together with equation (1), to solve the equation of balance of the weight of the column and the surface-tension forces at the boundaries solid body—liquid and gas (vacuum)—liquid:

$$G(\nu,\mu,h^*) = I_1(\nu,\mu,h^*) + I_{\mathrm{sl}}(h^*), \tag{2}$$

where

$$G(\nu, \mu, h^*) = \pi\rho g \int_0^{h^*} \left(F_{\text{l}}^2 - F_{\text{sl}}^2\right) dh;$$

$$I_{\text{sl}}(h^*) = 2\pi\alpha_{\text{sl}} \int_0^{h^*} \left(\frac{1}{2}\sqrt{F'^{\,2}_{\text{sl}h} + 1}\,\sin(2\operatorname{arc\,tg} F'_{\text{sl}}) + \frac{F_{\text{sl}}F''_{\text{sl}}\sin(\operatorname{arc\,tg} F'_{\text{sl}})}{1 + F'^{\,2}_{\text{sl}}}\right) dh;$$

$$I_{\text{l}}(\nu, \mu, h^*) = 2\pi\alpha_{\text{l}} \int_0^{h^*} \left(\frac{1}{2}\sqrt{F'^{\,2}_{\text{l}h} + 1}\,\sin(2\operatorname{arc\,tg} F'_{\text{l}}) + \frac{F_{\text{l}}F''_{\text{l}}\sin(\operatorname{arc\,tg} F'_{\text{l}})}{1 + F'^{\,2}_{\text{l}}}\right) dh.$$

Here the subscript "l" refers to the surface of separation liquid—gas (vacuum), and the subscript "sl" to the surface of separation solid body—liquid.

The boundary conditions are specified by the fact that the liquid always adjoins the surface of separation "sl" at the wetting angle θ:

$$\begin{aligned} &\text{for } h = h^* \quad \begin{cases} F'_{\text{l}h} = \operatorname{tg}\left[\operatorname{arc\,tg} F'_{\text{sl}h} + \theta\right], \\ F_{\text{l}}(h^*) = F_{\text{sl}}(h^*); \end{cases} \\ &\text{for } h = 0 \quad F'_{\text{l}h} = \infty. \end{aligned} \tag{3}$$

The last boundary condition is written for the case of transition of the column into the horizontal surface of the liquid ($^{8-11}$).

Solving equations (1) and (2) jointly, with the corresponding boundary conditions, one can determine those dimensions of the seed and of the melt (if it is used) for which the required size of the product being pulled is maintained. As is seen from conditions (3), this solution depends on the shape of the crystallization front, and the latter is determined by the thermal field in the product.

For the part of the volume of the cylindrical product being pulled that is bounded above and below by two isothermal surfaces corresponding to the temperatures T_1 and T_3, the heat-balance equation is valid:

$$2\pi\lambda_1 \int_0^{R_1} G_1(r)\, r\sqrt{F'^{\,2}_{1r} + 1}\, dr +$$

$$+ 2\pi \int_{h(T_3)}^{h(T_1)} \lambda_2(h) G_2(h) R_2(h) \sqrt{R'^{\,2}_{2h} + 1}\cos\left[\operatorname{arctg} R'_{2h} - \operatorname{arctg} R'_h(t_2(h))\right] dh =$$

$$= 2\pi\lambda_3 \int_0^{R_3} G_3(r)\, r\sqrt{F'^{\,2}_{3r} + 1}\, dr + \pi R_3^2 v\gamma H + \pi\frac{\partial}{\partial\tau}\int_{T_1}^{T_k} tc(t)\gamma(t)\frac{R^2(t,\tau)}{G_a(t,\tau)}\, dt, \tag{4}$$

where the equations of the isothermal surfaces are written in the form $h_i = F_i(r)$ or $r_i = R_i(h)$. The lower indices correspond to the surfaces bounding the volume under consideration (1—from above, 2—from the side, 3—from below). c is the heat capacity, γ the density, H the specific heat of crystallization, t the temperature, τ the time, G the temperature gradient, G_a the axial gradient, T fixed values of the temperature, and R fixed values of the radius.

In the simplest case, when the product being pulled has the form of a cylinder with constant diameter and lateral heat exchange is absent, this equation is considerably simplified. If equation (4) is applied under these conditions to the supercooling zone in the melt, we obtain

$$\lambda_l G_{al} + \gamma H v(\Delta t) = \lambda_s G_{as} + \Phi(\Delta t)\frac{\partial}{\partial \tau}\frac{1}{G_{al}} + \psi(T_\phi)\frac{\Delta \dot{t}}{G_{al}}, \tag{4a}$$

where $\psi(t) = tc(t)\gamma(t)R^2(t)$; $\Phi(\Delta t) = -\int_{T_\phi}^{T_k} \psi(t)\,dt$; the index l refers to the melt, and the index s to the solid material; $\Delta t = T_k - T_\phi$ is the supercooling at the given point of the crystallization front, T_k is the crystallization temperature, T_ϕ is the temperature at the front, and $v(\Delta t)$ is the dependence of the crystal growth rate on supercooling.

The differential equation describing the change with time of the amount of supercooling at the crystallization front has the form:

$$\frac{d(\Delta t)}{d\tau} = G_{al}(r,\tau)\,[\,n - v(\Delta t(\tau))\,], \tag{5}$$

where r is the spatial coordinate, n the rate of pulling of the product, and v the growth rate.

Thus, the growth process is described by the system of equations

$$\left.\begin{aligned} \Delta \dot{t} &= G'_{al}(n - v), \\ \frac{\Phi(\Delta t)}{G_{al}^2}\dot{G}_{al} - \lambda_l G_{al} + \frac{\psi(T_\phi)}{G_{al}}\Delta \dot{t} - \gamma H v(\Delta t) + \lambda_s G_{as} &= 0, \\ v &= v(\Delta t, C). \end{aligned}\right\} \tag{4 }$$

Here C is the vector defining the impurity content in the melt.

In the case where the drawn product is an anisotropic single crystal (for example, one having the structure of diamond), the internal heat exchange is described by the equation

$$\lambda_1 \frac{\partial^2 t}{\partial x_1^2} + \lambda_2 \frac{\partial^2 t}{\partial x_2^2} + \lambda_3 \frac{\partial^2 t}{\partial x_3^2} = c\gamma \frac{\partial t}{\partial \tau} + n \frac{\partial t}{\partial x_3} \tag{6}$$

with boundary conditions on the lateral surface of the crystal

$$\sigma \left[T^4(x_1, x_2, x_3, \tau) - T_e^4\right] = \lambda_1 \frac{\partial t}{\partial x_1}(x_1, x_2, x_3, \tau) + \lambda_2 \frac{\partial t}{\partial x_2}(x_1, x_2, x_3, \tau) \tag{7}$$

and at the crystallization front

$$q(x_1, x_2, x_3, \tau)\big|_{T_\phi} = \gamma H v(\Delta t(x_1, x_2, x_3, \tau)\big|_{T_\phi}) + \lambda_l G_{al}(x_1, x_2, x_3, \tau)\big|_{T_\phi}. \tag{8}$$

Here x_1, x_2, x_3 are three characteristic directions, the thermal conductivities along which are respectively $\lambda_1, \lambda_2, \lambda_3$.

For a complete mathematical description of the process it is also necessary to give equations describing the change in the concentration of impurities C at the crystallization front during growth (^{12}).

Until recently, the problem of controlling the drawing process was posed in the form of the requirement to maintain a prescribed product size $R_1(t)$ and to grow the product with a flat crystallization front, i.e. with $G_{rs} = 0$. The second requirement is explained by the desire to minimize the macroscopic density of dislocations in the crystal, which are the result of plastic deformations of the material under the action of thermal stresses.

However, the structure of the material is also influenced by axial temperature gradients in the crystal and, in addition, by the magnitude of supercooling at the crystallization front. In connection with this, the following problems of optimal control of single-crystal growth processes are posed.

The equation of internal heat exchange (6) is given with boundary conditions (7) and (8). Condition (8) is determined by the solution of the system of equations (46) and (5). The size of the growing crystal $R_1(t)$ is determined by equations (1), (2), (4) and conditions (3). The control actions are the temperature distribution $T_e(h)$ of concentric screens surrounding the drawn single crystal (in practical implementation these may be the heating powers of these screens in a resistance furnace or induction furnace), the power of the main furnace heaters surrounding the crucible with the melt, and also the drawing rate.

The control actions are constrained by restrictions on the modulus of the zeroth and first derivatives with respect to time and spatial coordinates, i.e.

$$|u_i| \le A_i; \qquad \left|\frac{\partial u_i}{\partial \tau}\right| \le B_i; \qquad \left|\frac{\partial u_i}{\partial h}\right| \le C_i; \qquad \left|\frac{\partial u_i}{\partial r}\right| \le D_i; \qquad i = 1, \dots, m, \tag{9}$$

where m is the total number of regulating actions.

The requirements on product quality reduce to constraints imposed on the function of the temperature gradients in the volume where the material is in a plastic state, $T_p \le t \le T_\phi$,

$$\int_0^\theta F\left[\theta, \int_V \Phi(G_a(r,h,\tau), G_r(r,h,\tau))\, dv\right] d\tau \tag{10}$$

and on the supercooling at the crystallization front Δt.

Problem 1. Under constraints (9)

$$\int_0^\theta F\left[\theta, \int_V \Phi(G_a(r,h,\tau),\, G_r(r,h,\tau))\, dv\right] d\tau \leqslant \delta_1, \tag{11}$$

$$\Delta t(r,h,\tau)_\Phi \leqslant \delta_2 \tag{12}$$

it is required to obtain the maximum productivity of the process

$$\Gamma(\theta) = \int_0^\theta R_1^2(\tau) n(\tau)\, d\tau = \max_u . \tag{13}$$

Problem 2. Under constraints (9) and (12), and also

$$\int_0^\theta R_1^2(\tau) n(\tau)\, d\tau \geqslant \delta_3 \tag{14}$$

it is required to obtain a product of the best quality

$$\int_0^\theta F\left[\theta, \int_V \Phi(G_a(r,h,\tau),\, G_r(r,h,\tau))\, dv\right] d\tau = \min_u . \tag{15}$$

The formulated problems can be solved on the basis of the theory of optimal control of objects with distributed parameters (13–15).

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Note: Figure translations are in progress. See original paper for figures.

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