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Abstract

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M. A. ROZENBLAT, Yu. D. ROZENTAL

THE OPTIMAL LAW OF REMAGNETIZATION OF FERROMAGNETIC CORES WITH A RECTANGULAR HYSTERESIS LOOP FROM THE STANDPOINT OF MINIMUM LOSSES

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As is known, a number of ferromagnetic devices in automation, telemechanics, and computing technology are based on a core with an almost rectangular hysteresis loop (r.h.l.), remagnetized from some initial state B_1 to the state B_2 during a prescribed time interval τ. In connection with this, the problem arises of determining the optimal remagnetization regime from the standpoint of minimum losses in the core. Reducing the losses in the core leads to an increase in the power gain and efficiency (in magnetic amplifiers), to an increase in the maximum frequency of clock pulses (in computing devices), to a reduction in the power consumed from the supply sources, etc.

The energy dissipated in the core during its switching process can be determined as

$$A = \int_0^\tau u(t)i(t)\,dt, \tag{1}$$

where $i(t)$ and $u(t)$ are, respectively, the current in the remagnetizing winding of the core and the e.m.f. induced in this winding:

$$i(t) = \frac{H(t)l}{w}, \tag{2}$$

$$u(t) = ws\frac{dB}{dt}, \tag{3}$$

where $H(t)$ is the instantaneous field value in the core averaged over the cross section; l is the length of the mean line of force; s is the cross-sectional area of the core; w is the number of turns of the winding.

Substituting (2) and (3) into (1), we obtain

$$A = v \int_0^{\tau} \frac{dB}{dt} H(t)\, dt, \tag{4}$$

where $v = sl$ is the volume of the core.

Depending on the type of device, one of the quantities $u(t) = k_1 B'$ or $i(t) = k_2 H(t)$ can usually be prescribed. Then the value of the other quantity is determined from the dynamic characteristics of the cores.

It has been established ([1–5]) that, for magnetic materials with an r.h.l., the dynamic characteristic of the core can be expressed in the following form:

$$B' = r(B)|H(t) - H_{\mathrm{st}}(B)| \tag{5}$$

or

$$H(t) = H_{\mathrm{st}}(B) + \frac{1}{r(B)} B', \tag{6}$$

where $r(B)$ is the reduced dynamic resistance; $H_{\mathrm{st}}(B)$ are the values of the field strength determined from the quasistatic hysteresis loop for the current value of B.

The form of the function $r(B)$ depends on the material and geometry of the core. For example, for a wide class of ferrites with a rectangular hysteresis loop one may take[6]

$$r(B) = r_m \left(1 - \frac{B^2}{B_s^2}\right), \tag{7}$$

where r_m is the maximum value of the reduced dynamic resistance.

For tape cores, whose dynamic properties are determined mainly by the influence of eddy currents, in accordance with (7) we have

$$r(B) = \frac{B_s}{k(B_s + B)}, \tag{8}$$

where k is a constant determined by the physical properties and geometry of the core.

Substituting (5) or (6) into (4), respectively, we obtain

$$A = v \int_0^{\tau} H(t)\,[H(t) - H_{\mathrm{st}}(B)]\, r(B)\, dt = v \int_0^{\tau} F_1[H(t), B]\, dt \tag{9}$$

or

$$A = v \int_0^\tau B' \left[H_{\text{st}}(B) + \frac{B'}{r(B)} \right] dt = v \int_0^\tau F_2(B, B')\, dt. \tag{10}$$

To find the optimal regime of remagnetization of the core it is more convenient to use (10), since the integrand in the latter is a function only of B and B':

$$F_2(B, B') = B' H_{\text{st}}(B) + \frac{(B')^2}{r(B)}, \tag{11}$$

whereas the integrand in (9) depends both on B and on $H(t)$. We note that $B' H_{\text{st}}(B)$ is the instantaneous value of hysteresis losses, and $(B')^2/r(B)$ is that of magnetic viscosity and/or eddy currents.

It is obvious that the problem of determining the minimum energy loss in this case can be solved by the usual methods of the calculus of variations[8] and consists in finding the solution of the Euler differential equation

$$\frac{\partial}{\partial B} F_2(B, B') - \frac{\partial}{\partial t} \left[\frac{\partial}{\partial B'} F_2(B, B') \right] = 0, \tag{12}$$

which determines the least value of the functional

$$T = \int_0^\tau F_2(B, B')\, dt \tag{13}$$

with respect to the sought function $B = B(t)$ (it should be noted that in each particular case the functions $B = B(t)$ and $F_2(B, B')$ must satisfy the basic requirements imposed on them when finding the extremal $B = B(t)$).

Since $F_2(B, B')$ does not contain t explicitly, the first integral of equation (12) has the form

$$F_2(B, B') - B' \frac{\partial}{\partial B'} F_2(B, B') = C.$$

Substituting (11), we obtain the following condition for minimum losses in remagnetizing the core:

$$\frac{(B')^2}{r(B)} = -C = C_1 = \text{const.} \tag{14}$$

Thus, the condition for the most economical remagnetization of a core is the constancy of the losses due to magnetic viscosity and eddy currents throughout the entire remagnetization process. It is characteristic that this condition does not depend on hysteresis losses, which for isotropic magnetic materials do not depend on the law of variation of the induction in time during a monotonic transition from B_1 to B_2.

Condition (14) can be used in the calculation and design of high-frequency and pulse devices of automation and computer technology using cores with a rectangular hysteresis loop.

Let us consider several examples of determining the optimal law of variation of the magnetic induction or field strength.

Suppose we have ferrite cores whose dynamic properties are described by equation (7). After substituting (7) into (14), we obtain

$$B' = \sqrt{C_1 r_m \left(1 - \frac{B^2}{B_s^2}\right)}$$

or, after integration,

$$\frac{B_s}{\sqrt{C_1 r_m}} \arcsin \frac{B}{B_s} = t + C_2.$$

The constants of integration C_1 and C_2 are readily determined from the boundary conditions:

$$\begin{aligned} &\text{at } t = 0 \qquad B = B_1, \\ &\text{at } t = \tau \qquad B = B_2. \end{aligned}$$

Finally, for remagnetization of a core along symmetric partial cycles of the hysteresis loop, we have

$$B = B_s \sin\left[\left(\frac{2\omega}{\pi} \arcsin \xi\right) t - \arcsin \xi\right], \tag{15}$$

$$u(t) = \frac{2B_s \omega s}{\pi} \arcsin \xi \cdot \cos\left[\left(\frac{2\omega}{\pi} \arcsin \xi\right) t - \arcsin \xi\right], \tag{16}$$

where $\xi = B_m / B_s$ is the modulation coefficient of the core.

After substituting the value found for B into (6) and taking (7) into account, we find, for the case under consideration, the optimal form of $H(t)$:

$$H(t) = H_{\text{st}}(B) + \frac{2B_s \omega \arcsin \xi}{r_m \pi \cos\left[\left(\frac{2\omega}{\pi} \arcsin \xi\right) t - \arcsin \xi\right]}. \tag{17}$$

When operating on the limiting hysteresis loop ($\xi \cong 1$), we accordingly have

$$B = -B_s \cos \omega t, \tag{18}$$

$$u(t) = B_s \omega s \sin \omega t = U_m \sin \omega t, \tag{19}$$

$$H(t) = H_{\rm st}(B) + \frac{B_s \omega}{r_m \sin \omega t}. \tag{20}$$

For tape cores, for which (8) is valid, when operating on the limiting hysteresis loop, as a result of solving the Euler equations (12), we obtain

$$B = 2B_s \left(\frac{\omega t}{\pi}\right)^{2/3} - B_s. \tag{21}$$

Analogously to what was said above, we determine the optimal forms of $u(t)$ and $H(t)$:

$$u(t) = \frac{4}{3}\omega s B_s \left(\frac{\omega}{\pi}\right)^{2/3} t^{-1/3}, \tag{22}$$

$$H(t) = \frac{8}{3} k B_s \left(\frac{\omega}{\pi}\right)^{4/3} t^{1/3} + H_{\rm st}(B). \tag{23}$$

Institute of Automation and Telemechanics

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Note: Figure translations are in progress. See original paper for figures.