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Abstract

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On the Choice of Criteria for the Synthesis of Combined Tracking Systems with Variable Structure

As shown in works ([1,3]), the use of combined tracking systems with variable structure makes it possible to improve substantially the static and dynamic accuracy of reproduction of a broad class of control actions when the parameters of the closed and open loops of the system vary within fairly wide limits. The basis for the synthesis of such systems is the choice of the region of existence of the sliding mode in the switching hyperplane. Let us consider the problem of choosing the region of existence of the sliding mode in the switching hyperplane of a combined tracking system with variable structure for a linear object of order n.

Let the behavior of a dynamical system in some region G of an n-dimensional phase space with coordinates $x_1, \dots, x_n$ be described by a system of nonhomogeneous differential equations with a discontinuous right-hand side

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{g}), \tag{1}$$

where

$$\mathbf{x} = (x_1, \dots, x_n); \qquad \mathbf{g} = (g_1, \dots, g_m), \quad m = n + 1;$$

$$\mathbf{f} = (f_1, \dots, f_n); \quad f_i = x_{i+1}, \quad i = 1, 2, \dots, n-1; \quad f_n = -\sum_{i=1}^{n} s_i x_i + \sum_{i=1}^{m} q_i^* g_i;$$

$$q_i^* = q_i - \psi_i(\mathbf{x}, \mathbf{g}), \quad i = 1, \dots, n; \qquad q_i = s_i, \quad i = 2, \dots, n; \quad q_{n+1} = 1;$$

$$\psi_i(\mathbf{x}, \mathbf{g}) = \begin{cases} a_i, & \text{when } g \sum_{j=1}^{n} c_{jx} j > 0, \\ a_i^*, & \text{when } g \sum_{j=1}^{n} c_{jx} j < 0. \end{cases}$$

If

$$g\sum_{j=1}^{n} c_{jx}j = 0,$$

then

$$\psi_i(\mathbf{x},\mathbf{g}) = a_i \quad \text{when } g\sum_{j=1}^{n} c_{jx}j \to +0,$$

$$\psi_i(\mathbf{x},\mathbf{g}) = a_i^* \quad \text{when } g\sum_{j=1}^{n} c_{jx}j \to -0.$$

It is known ($^{1-3}$) that in the sliding mode the solution of the original system (1) does not depend on $s_i, q_i, a_i, a_i^*, g(t)$. Here the solution of the original system coincides

with the solution of the linear system of differential equations

$$\frac{dx}{dt} = \mathbf{f}^0(\mathbf{x}), \tag{2}$$

where

$$\mathbf{x} = (x_1, \dots, x_n); \qquad \mathbf{f}^0 = (f_1^0, \dots, f_n^0); \qquad f_k^0 = x_{k+1}, \quad k = 1, \dots, n;$$

$$f_n^0 = \frac{1}{c_n}\sum_{k=1}^{n-1} c_{kx_{k+1}}, \qquad c_k \text{ are constants.}$$

The condition for the existence of a sliding mode in some region belonging to the switching hyperplane $c = \sum_{j=1}^{n} c_{jx}j = 0$ is given by the expression (?)

$$c\frac{dc}{dt} \leqslant 0. \tag{3}$$

The region of existence of the sliding mode is bounded by the intersections of the switching hyperplane with two hyperplanes in which the vector-functions are parallel to the switching hyperplane. According to (3), these intersections are determined by the following systems of equations:

$$\sum_{i=1}^{n} c_{ix}i = 0;$$

$$c_{ns}1x_1 - \sum_{i=2}^{n}(c_{i-1} - c_{ns}i)x_i = c_n\left[g_{n+1} + \sum_{i=1}^{n}(q_i - a_i)g_i\right] \tag{4}$$

and

$$\sum_{i=1}^{n} c_{ix}i = 0;$$

$$c_{ns}1x_1 - \sum_{i=2}^{n}(c_{i-1} - c_{ns}i)x_i = c_n\left[g_{n+1} + \sum_{i=1}^{n}(q_i - a_i^*)g_i\right]. \tag{5}$$

The systems of equations (4) and (5) determine two $(n-2)$-dimensional manifolds parallel to one another. Indeed, let us consider the system of equations that is the union of the systems of equations (4) and (5), writing it in matrix form:

$$\begin{bmatrix} c_1 & c_2 & \cdots & c_n \\ c_{ns}1 & -(c_1 - c_{ns}2) & \cdots & -(c_{n-1} - c_{ns}n) \\ c_{ns}1 & -(c_1 - c_{ns}2) & \cdots & -(c_{n-1} - c_{ns}n) \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} 0 \\ c_n\left[g_{n+1} + \sum_{i=1}^{n}(q_i - a_i)g_i\right] \\ c_n\left[g_{n+1} + \sum_{i=1}^{n}(q_i - a_i^*)g_i\right] \end{bmatrix}. \tag{6}$$

It turns out that the rank of the coefficient matrix with the appended column r_1 is greater than the rank of the coefficient matrix r; therefore the system is inconsistent, and the manifolds representing the solutions of the systems of equations (4) and (5) do not intersect one another. These manifolds determine the upper and lower boundaries of the region of existence of the sliding mode in the switching hyperplane c.

Let us estimate the magnitude of the region of existence of the sliding mode as a function of the parameters of the closed and open cycles, the type of disturbance, and the position of the switching hyperplane. As a measure for such an estimate we shall consider the algebraic sum of the lengths of two perpendiculars dropped from the origin to the intersection of the hyperplanes specified by the systems of equations (4) and (5). This sum may be regarded as the distance between the manifolds (4) and (5). The indicated perpendiculars lie on one straight line, since the manifolds (4) and (5) are parallel to one another, and this straight line itself lies in the switching hyperplane c, since at least two of its points are in this hyperplane.

The construction of both perpendiculars is carried out in the same way; therefore we shall restrict ourselves to considering the corresponding method of construction for only one of them. Obviously, the desired radius vector $\mathbf{l}(x_{1l}, \dots, x_{nl})$ is perpendicular to the single hyperplane L passing through the manifold formed by the intersection of the hyperplanes $c = 0$ and $c\,dx(a)/dt = 0$ (system of equations (4)). On the other hand, since the hyperplane L passes through the manifold (4), it must satisfy the equation of the pencil of hyperplanes passing through this manifold:

$$\sum_{i=1}^{n} A_i x_i + D + \lambda \sum_{i}^{n} c_i x_i = 0, \tag{7}$$

where $A_i = -(c_{i-1} - c_n s_i)$, $c_0 = 0$, $D = -c_n \left[g_{n+1} + \sum_{i=1}^{n} (q_i - a_i) g_i\right]$. In addition, the hyperplane L is perpendicular to the switching hyperplane c, in which the desired radius vector $\mathbf{l}$ lies. From the condition of perpendicularity of two hyperplanes,

$$\sum_{j=1}^{n} c_j (\lambda c_j + A_j) = 0, \tag{8}$$

we find the value of the coefficient λ satisfying the equation of the hyperplane L:

$$\lambda = -\sum_{j=1}^{n} A_j c_j \Big/ \sum_{j=1}^{n} c_j^2. \tag{9}$$

Hence the equation of the hyperplane L is written in the form

$$\sum_{i=1}^{n} \left(A_i - \left(\sum_{j=1}^{n} A_j c_j \Big/ \sum_{j=1}^{n} c_j^2 \right) c_i \right) x_i + D = 0. \tag{10}$$

The length of the desired radius vector is determined by the free term in equation (10), transformed to normal form, i.e.,

$$\delta_l = \frac{-D}{\left\{ \sum_{i=1}^{n} \left[A_i - \left(\sum_{j=1}^{n} A_j c_j \Big/ \sum_{j=1}^{n} c_j^2 \right) c_i \right]^2 \right\}^{1/2}}. \tag{11}$$

Substituting here the values of D and A_i from (7) and putting $c_n = 1$, which does not impair the generality of the reasoning, we finally obtain

$$\delta_l = \left[g_{n+1} + \sum_{i=1}^{n} (q_i - a_i) g_i \right] \frac{\sum_{j=1}^{n} c_j^2}{\left\{ \sum_{i=1}^{n} \left[(c_{i-1} - s_i) \sum_{j=1}^{n} c_j^2 - c_i \sum_{j=1}^{n} c_j (c_{j-1} - s_j) \right]^2 \right\}^{1/2}}. \tag{12}$$

In an analogous manner one determines the length of the radius vector $\mathbf{m}(x_{1m}, \dots, x_{nm})$, perpendicular to the intersection of the hyperplanes $c = 0$ and $c\,dx(a^*)/dt = 0$,

contained in the hyperplane M:

$$\delta_m = \left[g_{n+1} + \sum_{i=1}^{n}(q_i - a_i^*)g_i\right] \times$$

$$\times \frac{\sum_{j=1}^{n} c_j^2}{\left\{\sum_{i=1}^{n}\left[(c_{i-1} - s_i)\sum_{j=1}^{n} c_j^2 - c_i \sum_{j=1}^{n} c_j(c_{j-1} - s_j)\right]^2\right\}^{1/2}}. \qquad (13)$$

Then the distance between the intersections of the hyperplanes (4) and (5) can be determined as

$$d = |\delta_l - \delta_m| =$$

$$= \left| \sum_{i=1}^{n}(a_i - a_i^*)g_i \frac{\sum_{j=1}^{n} c_j^2}{\left\{\sum_{i=1}^{n}\left[(c_{i-1} - s_i)\sum_{j=1}^{n} c_j^2 - c_i \sum_{j=1}^{n} c_j(c_{j-1} - s_j)\right]^2\right\}^{1/2}} \right|. \qquad (14)$$

As is seen from equations (12), (13), and (14), the selection of the region of existence of the sliding mode in the switching hyperplane of a combined tracking system can be carried out separately depending on: 1) the position of the switching hyperplane and the parameters of the closed loop; 2) the form of the reproduced control actions and the parameters of the open loop. Thus, the choice of the position of the switching hyperplane that is optimal from the standpoint of the magnitude of the region of existence of the sliding mode is determined by the function

$$\beta_n = \frac{\sum_{j=1}^{n} c_j^2}{\left\{\sum_{i=1}^{n}\left[(c_{i-1} - s_i)\sum_{j=1}^{n} c_j^2 - c_i \sum_{j=1}^{n} c_j(c_{j-1} - s_j)\right]^2\right\}^{1/2}}, \qquad (15)$$

and the choice of the parameters of the open loop is determined by the functions

$$\alpha_n = g_{n+1} + \sum_{i=1}^{n} (q_i - a_i) g_i, \tag{16}$$

$$\alpha_n^* = g_{n+1} + \sum_{i+1}^{n} (q_i - a_i^*) g_i. \tag{17}$$

Then equations (12)–(14) will be written, respectively, as

$$\delta_l = \alpha_n \beta_n; \tag{18}$$

$$\delta_m = \alpha_n^* \beta_n; \tag{19}$$

$$d = (\alpha_n - \alpha_n^*) \beta_n. \tag{20}$$

Thus, in order to obtain the maximum magnitude of the region of existence of the sliding mode, it is necessary to require that the functions α_n, α_n^*, and β_n be maximal.

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Note: Figure translations are in progress. See original paper for figures.