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# PHYSICS

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**Abstract**

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PHYSICS

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# ON LOW-TEMPERATURE EXPANSIONS IN THE THEORY OF FERROMAGNETISM

*(Presented by Academician N. N. Bogolyubov, 31 X 1962)*

We shall consider here the problem of determining the spectrum of elementary excitations and the magnetization of an isotropic ferromagnetic dielectric in the low-temperature region. For simplicity we shall restrict ourselves to the case of spin $S = 1$ (in units of $\hbar/2$) and a simple cubic lattice; the interaction will be taken into account in the nearest-neighbor approximation.

It is known that at sufficiently low temperatures the spectrum of elementary excitations of such a ferromagnet can be represented as a set of so-called spin waves, which may approximately be regarded as noninteracting with one another ([1]). In this case, for the magnetization per atom one obtains an expression of the form:

$$\sigma_{Bl} = 1 - 2Z\left(^3/_2, x\right)\tau^{3/2} - \frac{3\pi}{2}Z\left(^5/_2, x\right)\tau^{5/2} - \frac{33\pi^2}{16}Z\left(^7/_2, x\right)\tau^{7/2} - \dots, \quad (1)$$

where

$$\tau = \frac{\vartheta}{8\pi I}; \qquad Z(p, x) = \sum_{n=1}^{\infty} n^{-p}\exp(-nx); \qquad x = \frac{\mu_B H}{3I}, \quad (2)$$

$\vartheta$ is the temperature in units of $kT$; $Z(p, 0) = \xi(p)$.

To construct subsequent approximations to this solution, it is necessary to take into account both the dynamic interaction of spin waves caused by the corresponding terms of the Hamiltonian, and the kinematic interaction arising from restrictions on the occupation numbers of spin waves and connected with the fact that the spin at a given lattice site can take only a finite number of values ([2]). Attempts to take this interaction into account, undertaken in a number of works ([2–7]), led to the appearance of additional terms in powers of temperature in expansion (1), and different temperature-power corrections were obtained in different papers. In the works ([2]), which are considered the most consistent, it was found that the corrections from the dynamic and kinematic interactions

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compensate one another to a high degree of accuracy and give a contribution to expansion (1) only beginning with terms of order $\tau^{8/2}$:

$$\sigma_D = \sigma_{Bl} - 6\pi Z\left({}^3/_2, x\right) Z\left({}^5/_2, x\right) \tau^{8/2} - ... \tag{3}$$

Later the results of works ([2]) were reproduced by other methods in papers ([8,9]).

Let us note that, in the calculations in works ([2,8]), the Hamiltonian of the real spin system is replaced by a certain equivalent operator in Bose operators. Application of the method of two-time temperature Green' s functions for calculating the magnetization with these Hamiltonians ([10,11]) also leads to expansion (3). Since the transition from spin operators with the real Hamiltonian to Bose operators leads to well-known difficulties in taking into account the restrictions on the occupation numbers of spin waves, we shall here consider once more the problem of calculating the magnetization at low temperatures. In doing so, we shall use Pauli operators, which are fully equivalent to the initial spin operators. We shall calculate the magnetization by means of two-time temperature Green' s functions ([12]), using perturbation theory in the spirit of work ([13]) to determine the latter.

Let the Hamiltonian of the system have the form:

$$\mathscr{H} = \mathcal{E}_0 + (2\mu_{BH} + 2J(0)) \sum n_f - \sum 2I(f_1 - f_2) b^+_{f_1} b_{f_2} - \varepsilon \sum 2I(f_1 - f_2) n_{f_1} n_{f_2}, \tag{4}$$

where

$$\mathcal{E}_0 = -\frac{1}{2} N J(0) - N\mu_{BH}; \qquad \sum_{(f)} 1 = N; \qquad I(0) = 0;$$

$$I(f) = \frac{1}{N} \sum_{(\nu)} J(\nu) e^{-i(f,\nu)}; \qquad J(\nu) = \sum_{(f)} I(f) e^{i(f,\nu)}; \tag{5}$$

$f$ is the number of a lattice site, $N$ is the number of sites in it, $b_f, b^+_f$ are Pauli operators:

$$b^+_{fb_{f'}} - b^+_{f'} b_f = \Delta(f - f')(1 - 2\varepsilon n_f); \qquad b_{fb_{f'}} - b_{f'} b_f = 0;$$

$$b^2_f = 0; \qquad n_f = b^+_f b_f (= 0;\, 1); \tag{6}$$

$\varepsilon$ is a formal small parameter.

The equations of motion for the operators $b_f$ have the form

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$$i\frac{db_f}{dt} = (2\mu_{BH} + 2J(0))b_f - \sum 2I(f-f')b_{f'} + \varepsilon \sum 4I(f-f')n_{fb_{f'}} -$$

$$-\varepsilon \sum 4I(f-f')b_{fn_{f'}}; \tag{7}$$

here the third term corresponds to the kinematic interaction of spin waves, and the fourth to the dynamic one. Following paper [12], we introduce the Green' s functions

$$G_1 = \langle\langle b_f \mid b_g^+ \rangle\rangle; \qquad G_2 = \langle\langle b_{f_1}^+ b_{f_2} \, \varepsilon_{f_3} \mid b_g^+ \rangle\rangle; \quad G_3 = \langle\langle b_{f_1}^+ b_{f_2} b_{f_3} b_{g_1}^+ n_{g_2} \mid b_g^+ n_{g_1} \rangle\rangle; \ldots \tag{8}$$

and, to determine them, construct a chain of coupled equations, differentiating $G_1$ with respect to $t$, $G_2$ with respect to $t'$, $G_3$ with respect to $t, \ldots$ We shall regard the dynamic and kinematic interaction as a small perturbation.

As is not difficult to see, with such an introduction of the small parameter $\varepsilon$, breaking off the chain of equations for the functions $G$ according to the smallness of the perturbation corresponds to discarding functions with higher Green' s functions. The "small parameter" $\varepsilon$ is introduced into formulas (4), (6), (7) so as to take this circumstance into account.

Writing out the equations for the functions $G_1, G_2, G_3$ accurate up to quantities of order $\varepsilon$ inclusive, we construct, according to the scheme of paper [13], the mass operator for the function $G_1$. For this purpose we pass to Fourier images in time and coordinates:

$$\langle\langle b_f \mid b_g^+ \rangle\rangle = \frac{1}{N} \sum_{(\mu)} e^{i(g-f,\mu)} G_\mu. \tag{9}$$

The equation of the second approximation for the function $G_\mu$ has the form

$$(E - E_\mu - M_\mu(E))G_\mu = \frac{i\sigma}{2\pi}, \tag{10}$$

where $\sigma$ is the relative magnetization per site, $E_\mu$ is the Bloch energy of a spin wave with wave vector $\mu$, and $M_\mu(E)$ is the mass operator:

$$\sigma = 1 - 2\varepsilon\langle n_f \rangle = 1 - 2\varepsilon\bar{n}; \qquad E_\mu = 2\mu_{BH} + 2J(0) - 2J(\mu);$$

$$M_\mu(E) = \frac{\varepsilon}{\sigma N} \sum 4I(f_1 - f_2) \left(e^{i(f_1,\mu)} - e^{i(f_2,\mu)}\right) e^{-i(p,\mu)} \times$$

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$$\times\left[\langle n_{f_1} b_{f_2} b_p^+\rangle - \langle b_p^+ n_{f_1} b_{f_2}\rangle\right] +$$

$$+\frac{\varepsilon^2}{\sigma N^3}\sum[4J(\mu+\nu)-4J(\nu)]\,4I(g_1-g_2)\left(e^{-i(g_1,\mu)}-e^{-i(g_2,\mu)}\right)\times$$

$$\times\exp[-i(p_1,\mu_1)+i(p_2,\mu_1-\nu)+i(p_3,\mu+\nu)]\times$$

$$\times\frac{\langle b_{p_1}^+ b_{p_2} b_{p_3}^+ b_{g_2} n_{g_1}\rangle - \langle b_{g_2}^+ n_{g_1} b_{p_1}^+ b_{p_2} b_{p_3}\rangle}{E+E_{\mu_1}-E_{\mu_1-\nu}-E_{\mu+\nu}} - \left\{\frac{\varepsilon}{\sigma N}\sum 4I(f_1-f_2)\,...\right\}^2 (E-E_\mu)^{-1}. \tag{11}$$

Next we determine the commutators entering this expression from the solution of the previously indicated approximate system of equations for the functions $G$, and restrict ourselves to the lowest-order terms in temperature. As a result we obtain:

$$M_\mu(E) \simeq \frac{4\varepsilon}{N}\sum_{(\nu)}[J(\mu)+J(\nu)-J(0)-J(\mu-\nu)]\,\overline{N}_\nu+\frac{8\varepsilon^2}{N^2}\sum_{(\nu_1,\nu_2)}[2J(\mu+\nu_1)-2J(\nu_1)]\frac{J(\nu_1-\nu_2)+J(\mu+\nu_1)-J(\mu-}{E+E_{\nu_2}-E_{\nu_1-\nu_2}-} \tag{12}$$

where

$$\overline{N}_\nu = \left[\exp\left(\frac{\mathcal{E}_\nu}{\vartheta}\right)-1\right]^{-1}, \tag{13}$$

$\mathcal{E}_\nu$ is the energy of one-particle excitations (the pole of the function $G_\mu$).

Let us note that the numbers $\overline{N}_\nu$ are related to the mean occupation numbers of spin waves $\langle b_\nu^+ b_\nu\rangle$ and of spins $\langle n_f\rangle$ by the formulas:

$$\langle b_\nu^+ b_\nu\rangle = \sigma\overline{N}_\nu; \qquad \overline{n} = \langle n_f\rangle = \frac{\sigma}{N}\sum_{(\nu)}\overline{N}_\nu; \qquad \langle b_g^+ b_f\rangle = \frac{\sigma}{N}\sum_{(\nu)} e^{i(g-f,\nu)}\overline{N}_\nu. \tag{14}$$

The real ($M'_\mu$) and imaginary ($M''_\mu$) parts of the mass operator at $E$ satisfying the equation

$$E - E_\mu - M'_\mu(E) = 0,$$

determine the corrections to the spin-wave energy $E_\mu$ and their lifetime. In the nearest-neighbor approximation and for $H = 0$:

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$$M'_\mu \simeq -\varepsilon \left(1+\varepsilon\frac{2a}{z}\right) 4Iz(1-\gamma_\mu)\frac{\pi^\nu}{\delta^3}\zeta\left(\frac{5}{2}\right)\tau^{5/2};$$

$$M''_\mu \simeq \varepsilon^2 4Iz(1-\gamma_\mu)\frac{8\nu^2}{36\delta^6}\zeta(3)\tau^{6/2}; \tag{15}$$

where

$$\tau = \frac{3\vartheta}{4\pi Iz}; \qquad a = \frac{1}{N}\sum_{(\nu)}\frac{1+2\gamma_\nu}{1-\gamma_\nu}; \qquad \gamma_\mu = \frac{1}{z}\sum_{(\delta)} e^{i(\mu,\delta)}, \tag{16}$$

$z$ is the number of nearest neighbors, $\delta$ is the distance between them, $\zeta(p)$ is the Riemann zeta-function, and approximately $a \sim 2$.

The expressions obtained for $M'_\mu$, $M''_\mu$ correspond to the results of papers ([2],[8] ). Indeed, if one uses the method set out for the spin-wave Hamiltonians of these papers, then one obtains ([10]) for $M_\mu$, $M''_\mu$ expressions coinciding with (15).

Thus, in calculating the spectrum of elementary excitations and their lifetime, the result does not depend on which representation of the spin Hamiltonian is used: through Pauli operators or through Bose operators (with one or another account of the kinematic interaction). The situation changes when one passes to the calculation of the magnetization. In calculating the magnetization in Bose operators we obtain

$$\sigma = 1 - 2\varepsilon\langle n_f\rangle = 1 - \frac{2\varepsilon}{N}\sum_{(\nu)}\overline{N}_\nu; \tag{17}$$

when calculating in Pauli operators, according to formulas (14), we obtain

$$\sigma = 1 - 2\varepsilon\langle n_f\rangle = 1 - \frac{2\varepsilon\sigma}{N}\sum_{(\nu)}\overline{N}_\nu. \tag{18}$$

Expression (17) leads, as is easy to see, to the power expansion (3), in which the nontrivial corrections begin with terms of order $\tau^{8/2}$. Expression (18) leads to the expansion:

$$\sigma = \sigma_{Bl} + 4\zeta^2(3/2)\tau^{6/2} + O(\tau^{9/2}) \tag{19}$$

with nontrivial corrections in terms of order $\tau^{9/2}$. This discrepancy is apparently connected with an insufficient allowance for the kinematic interaction when representing the magnetization through averages of Bose operators.

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*Note: Figure translations are in progress. See original paper for figures.*

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