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S. V. EMEL'YANOV, V. I. UTKIN

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Abstract

Full Text

CYBERNETICS AND CONTROL THEORY

S. V. EMEL' YANOV, V. I. UTKIN

APPLICATION OF AUTOMATIC CONTROL SYSTEMS WITH VARIABLE STRUCTURE FOR CONTROLLING OBJECTS WHOSE PARAMETERS VARY OVER WIDE LIMITS

(Presented by Academician B. N. Petrov, 27 III 1963)

In a number of cases, the use of automatic control systems with variable structure makes it possible to considerably weaken the influence of changes in the parameters of the object on the quality of the control process. As shown in work ⁽¹⁾, in the region G of an n -dimensional phase space x_1, \dots, x_n there exists a certain region $U \subset S$ (S is a hyperplane defined by the equation

$$\sum_{i=1}^n c_i x_i = 0),$$

the motion in which is determined only by the coefficients c_i .

However, as follows from ⁽²⁾, the boundaries of the region U depend both on c_i and on the parameters of the controlled object. It may happen that, during motion in the region U , the representative point reaches the boundary of this region and leaves the hyperplane S . Then the further motion of the system will be determined not only by the coefficients c_i , but also by the variable parameters of the object. In order that the solution of the differential equation of motion after the representative point enters S should not depend on changes, within certain limits, of the coefficients of the original system of differential equations, the region U for these values of the coefficients must coincide with the hyperplane S ($U \supseteq S$).

Fig. 1

Let, in the region G of the n -dimensional space x_1, \dots, x_n , the motion of a dynamical system be described by the system of differential equations

$$\frac{d\bar{x}}{dt} = \bar{f}(\bar{x}, \bar{\psi}), \tag{1}$$

where

$$\bar{x} = (x_1, \dots, x_n); \quad \bar{\psi} = (\psi_1, \dots, \psi_{n-1}); \quad \bar{f} = (f_1, \dots, f_n); \quad f_i = x_{i+1}$$

$$(i = 1, 2, \dots, n-1); \quad f_n = -\sum_{i=1}^n a_i x_i - \sum_{i=1}^{n-1} \psi_i(\bar{x}) x_i;$$

$$\psi_i(\bar{x}) = \begin{cases} \omega_i & \text{for } gx_i > 0, \\ \lambda_i & \text{for } gx_i < 0^*, \end{cases} \quad (i = 1, 2, \dots, n-1) \quad (2)$$

$$g = \sum_{i=1}^n c_i x_i;$$

ω_i, λ_i, c_i are constants, $c_n = 1$; a_i are coefficients varying with time.

The region U on the hyperplane S , according to (2), is determined by the relations

$$\bar{c} \frac{d\bar{x}}{dt} < 0 \quad \text{for } g > 0,$$

$$\bar{c} \frac{d\bar{x}}{dt} > 0 \quad \text{for } g < 0, \quad (3)$$

where $c = (c_1, \dots, c_n)$.

From (3) and (1) we obtain

$$\sum_{i=1}^{n-1} (c_{i-1} - \psi_i(\bar{x}) - a_i) x_i + (c_{n-1} - a_n) x_n < 0 \quad \text{for } g > 0,$$

$$\sum_{i=1}^{n-1} (c_{i-1} - \psi_i(\bar{x}) - a_i) x_i + (c_{n-1} - a_n) x_n > 0 \quad \text{for } g < 0,$$

$$c_0 = 0. \quad (4)$$

The control vector $\bar{\psi}$ must ensure that conditions (4) are satisfied for any point

$$\left(x_1, x_2, \dots, x_{n-1}, -\sum_{i=1}^{n-1} c_i x_i \right)$$

of the hyperplane S , i.e.,

$$\sum_{i=1}^{n-1} (c_{i-1} - \psi_i(\bar{x}) - a_i - c_{n-1}c_i + a_n c_i)x_i < 0 \quad \text{for } g > 0,$$

$$\sum_{i=1}^{n-1} (c_{i-1} - \psi_i(\bar{x}) - a_i - c_{n-1}c_i + a_n c_i)x_i > 0 \quad \text{for } g < 0. \quad (5)$$

From expressions (2) and (5) we obtain the necessary and sufficient conditions for the coincidence of the region U with the hyperplane S ($U \supseteq S$) under variation, within a certain range, of the coefficients a_i :

$$\omega_i > \sup_{a_i, a_n} (c_{i-1} - a_i - c_{n-1}c_i + a_n c_i),$$

$$\lambda_i < \inf_{a_i, a_n} (c_{i-1} - a_i - c_{n-1}c_i + a_n c_i). \quad (i = 1, \dots, n-1) \quad (6)$$

Thus, if the representative point falls on the hyperplane S and ω_i and λ_i satisfy (6), then the subsequent motion will not depend on a_i , but is determined by the coefficients c_i .

It should be noted that the conditions $U \supseteq S$ for constant values of the object parameters, derived in (2), can be obtained if one takes ω_i and λ_i , for $i = 2, \dots, n-1$, in (6) equal to zero.

* In the case $gx_i = 0$

$$\psi_i(\bar{x}) = \omega_i \quad \text{for } gx_i \rightarrow +0; \quad \psi_i(\bar{x}) = \lambda_i \quad \text{for } gx_i \rightarrow -0.$$

Example. Let the automatic control system be described by the system of differential equations

$$\begin{aligned} \frac{dx_1}{dt} &= x_2, \\ \frac{dx_2}{dt} &= x_3, \\ \frac{dx_3}{dt} &= -(a_1 x_1 + a_2 x_2 + a_3 x_3) - \psi_1(x_1, x_2, x_3)x_1 - \psi_2(x_1, x_2, x_3)x_2, \end{aligned} \quad (7)$$

where

$$\psi_1(x_1, x_2, x_3) = \begin{cases} \omega_1 & \text{for } gx_1 > 0, \\ \lambda_1 & \text{for } gx_1 < 0^*; \end{cases}$$

$$\psi_2(x_1, x_2, x_3) = \begin{cases} \omega_2 & \text{for } gx_2 > 0, \\ \lambda_2 & \text{for } gx_2 < 0^{**}; \end{cases}$$

$$g = c_1x_1 + c_2x_2 + x_3;$$

a_1, a_2, a_3 are variable coefficients varying within the limits

$$a_{1\min} < a_1 < a_{1\max},$$

$$a_{2\min} < a_2 < a_{2\max},$$

$$a_{3\min} < a_3 < a_{3\max};$$

c_1, c_2 are constant positive quantities.

For the condition $U \subseteq S$, according to (6), to be fulfilled, it is necessary and sufficient that

$$\begin{aligned} \omega_1 &> -a_{1\min} - c_2c_1 + a_{3\max}c_1, \\ \lambda_1 &< -a_{1\max} - c_2c_1 + a_{3\min}c_1, \\ \omega_2 &> c_1 - a_{2\min} - c_2^2 + a_{3\max}c_2, \\ \lambda_2 &< c_1 - a_{2\max} - c_2^2 + a_{3\min}c_2. \end{aligned} \quad (8)$$

Conditions (3) for the system under consideration have the form

$$\gamma_1 = (-\psi_1 - a_1)x_1 + (c_1 - \psi_2 - a_2)x_2 + (c_2 - a_3)x_3 < 0 \quad \text{for } g > 0,$$

$$\gamma_2 = (-\psi_1 - a_1)x_1 + (c_1 - \psi_2 - a_2)x_2 + (c_2 - a_3)x_3 > 0 \quad \text{for } g > 0. \quad (9)$$

When (8) is satisfied, the plane S and the surfaces γ_1 and γ_2 (Fig. 1) have no common points, except for the origin, if the coefficients a_1, a_2 , and a_3 vary within the indicated range.

Institute of Automation and Telemechanics

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2. S. V. Emel' yanov, V. A. Taran, Izv. AN SSSR, Energetika i avtomatika, No. 3 (1962).

* In the case $gx_1 = 0$

$$\psi_1(x_1, x_2, x_3) = \omega_1 \text{ for } gx_1 \rightarrow +0; \quad \psi_1(x_1, x_2, x_3) = \lambda_1 \text{ for } gx_1 \rightarrow -0.$$

** In the case $gx_2 = 0$

$$\psi_2(x_1, x_2, x_3) = \omega_2 \text{ for } gx_1 \rightarrow +0; \quad \psi_2(x_1, x_2, x_3) = \lambda_2 \text{ for } gx_2 \rightarrow -0.$$

Note: Figure translations are in progress. See original paper for figures.

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