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Abstract

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QUANTUM-STATISTICAL THEORY OF FERROMAGNETIC RESONANCE

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The behavior of the magnetization of a ferromagnet under conditions of magnetic resonance has a qualitatively different character in the temperature intervals $T \gg T_K$ and $T \ll T_K$, where T_K is the Curie temperature. As experimental data show ([1]), for $T \gg T_K$ the relaxation of the magnetization is described by equations of the Bloch type with equal longitudinal and transverse relaxation times:

$$\partial \mathbf{M}/\partial t = \gamma[\mathbf{MH}] - (\mathbf{M} - \chi_0 \mathbf{H})/T, \tag{1}$$

where $\mathbf{M}$ is the magnetization, $\mathbf{H}$ is the external field, γ is the magnetomechanical ratio, χ_0 is the static susceptibility, and T is the spin-spin relaxation time. A microscopic derivation of equation (1) was obtained earlier by the authors, and the theoretical results agree well with experiment ([2]).

In the region of low temperatures ($T \ll T_K$), several phenomenological equations proposed by various authors ([3–8]) are used in the literature to describe ferromagnetic resonance. These equations are in fact various modifications of the equation first obtained by L. D. Landau and E. M. Lifshitz ([3]), which has the form

$$\partial \mathbf{M}/\partial t = \gamma[\mathbf{MH}] - \lambda[\mathbf{M}[\mathbf{MH}]]/M^2. \tag{2}$$

Equation (2) is in qualitative agreement with experimental data, although some experimental results cannot be explained by the above phenomenological equations (for example, the difference between the observed values of the g-factor and its theoretical value ([9,10])).

In the present work we shall set forth a quantum-statistical derivation of the equations of ferromagnetic resonance, valid practically over the entire temperature interval, and for $T \gg T_K$ these equations reduce to equations (1). We shall consider a spherical isotropic ferromagnet with purely spin magnetism, on which, in addition to a homogeneous external constant field $\mathbf{H}_0$, directed along the z-axis and bringing the specimen to saturation, there acts a homogeneous alternating field $\mathbf{H}_1(t)$. The case of homogeneous precession of the magnetization is considered, which occurs for values of $\mathbf{H}_1$ smaller than a certain critical value ([11]).

Our problem consists in obtaining the equations of ferromagnetic resonance from the rigorous equation for the density matrix, which in the case under consideration has the form:

$$\hbar\, \partial\rho(t)/\partial t = -i[-\mu\mathbf{H}(t)\hat{\mathbf{S}} + \hat{H}_{\text{ex}} + \hat{H}_{\text{dip}}, \rho(t)], \tag{3}$$

where $\mathbf{H}(t)$ is the external magnetic field (the constant $\mathbf{H}_0$ and the alternating $\mathbf{H}_1(t)$),

$$\hat{H}_{\text{ex}} = -\frac{1}{2}\sum_{i>k} I_{ik}\hat{\mathbf{S}}_i\hat{\mathbf{S}}_k; \qquad \hat{H}_{\text{dip}} = \mu^2\sum_{i>k}\left[\frac{\hat{\mathbf{S}}_i\hat{\mathbf{S}}_k}{r_{ik}^3} - 3\frac{(\hat{\mathbf{S}}_i\mathbf{r}_{ik})(\hat{\mathbf{S}}_k\mathbf{r}_{ik})}{r_{ik}^5}\right].$$

Here $\hat{S}$ is the operator of the total spin of the system; $\hat{S}_i$ is the spin operator located at the i-th site of the crystal lattice; $\mathbf{r}_{ik}$ is the vector connecting the i-th and k-th lattice sites; I_{ik} is the exchange integral; $\mu = \gamma\hbar$, and the square brackets denote a commutator.

We consider the case in which $H_{\text{ex}} \gg H_{\text{dip}}$ and $\mu H(t)S$, where H_{ex} and H_{dip} are the mean values of the exchange and dipole interactions, respectively. In equation (3) the interaction of the spin system with the lattice has not been taken into account, since spin-lattice relaxation apparently plays a secondary role in the process of ferromagnetic resonance ([12]). The influence of the lattice is manifested in the fact that it maintains an unchanged temperature of the exchange interaction, whereas the character of the relaxation is determined by the interaction of the external field with the exchange reservoir by means of the dipole interaction.

The basis of the following reasoning is the method first developed by one of the authors ([13]). In equation (1), let us pass to the interaction representation

$$\rho(t) = \exp i\hbar^{-1}\left\{\int_0^t \mu H(\tau)\hat{S}\, d\tau - \hat{H}_{\text{ex}}t\right\}\rho'(t)\exp i\hbar^{-1}\left\{-\int_0^t \mu H(\tau)\hat{S}\, d\tau + \hat{H}_{\text{ex}}t\right\}. \tag{4}$$

For $\rho'(t)$ we obtain:

$$\hbar\, \partial\rho'(t)/\partial t = -i[\hat{H}_{\text{dip}}(t), \rho'(t)],$$

$$\hat{H}_{\text{dip}}(t) = \exp i\hbar^{-1}\left\{-\int_0^t \mu H(\tau)\hat{S}\,d\tau + \hat{H}_{\text{ex}}t\right\}\hat{H}_{\text{dip}}\times$$

$$\times\exp i\hbar^{-1}\left\{\int_0^t \mu H(\tau)\hat{S}\,d\tau - \hat{H}_{\text{ex}}t\right\}.$$

Let us split the density matrix $\rho'(t)$ into two parts, $\rho'(t) = \rho'_1(t) + \rho'_2(t)$, where $\rho'_1(t)$ is the part of the density matrix $\rho'(t)$ that is diagonal in the representation in which $\hat{H}_{\text{ex}}$ and the operator of the projection of the total spin onto the direction of the instantaneous magnetization are simultaneously diagonal. In ([2]) it was shown that $\rho'_1(t)$ and $\rho'_2(t)$ are related by the relation

$$\rho'_2(t) = -i\hbar^{-1}\int_0^t dt'\,[\hat{H}_{\text{dip}}(t'), \rho'_1(t')],$$

where $\rho'_1(t)$ has the form

$$\rho'_1(t) = C\exp\{\alpha(t)\hat{S}_z + \beta(t)\hat{S}_x + \gamma(t)\hat{S}_y + \delta(t)\hat{H}_{\text{ex}}\};$$

C is a normalization constant.

Returning, with the aid of (4), from the interaction representation to the laboratory system, we find:

$$\rho_2(t) = -i\hbar^{-1}\int_0^t dt'\,[\hat{H}_{\text{dip}}(t,t'), \rho'_1(t')], \tag{5}$$

$$\hat{H}_{\text{dip}}(t,t') = \exp i\hbar^{-1}\left\{\int_{t'}^t \mu H(\tau)\hat{S}\,d\tau - \hat{H}_{\text{ex}}(t-t')\right\}\hat{H}_{\text{dip}}\times$$

$$\times\exp i\hbar^{-1}\left\{-\int_{t'}^t \mu H(\tau)\hat{S}\,d\tau + \hat{H}_{\text{ex}}(t-t')\right\},$$

where $\rho_1(t)$ evidently has the same form as before.

Taking into account that the correlation time in the system under consideration is equal to $\sim \hbar/H_{\text{ex}}$ [2], we note that the integration in (5) is in fact carried out over the interval $\Delta t' = \hbar/H_{\text{ex}}$. Since $\rho_1(t)$ changes only weakly over such an interval (the relaxation times are much greater than $\hbar/H_{\text{ex}}$), $\rho_1(t')$ may be replaced by $\rho_1(t)$ [2]. For the same reason, the exponential factors containing integrals over τ may be expanded in a series (since $\mu H(t)S \ll H_{\text{ex}}$), and the integrals over τ themselves may be represented in the form $\mu H(t)\hat{S}(t-t')$ (since the external field changes little over times of order $\hbar/H_{\text{ex}}$). Moreover, the integration over t' can obviously be extended to infinity.

Taking the above into account, we obtain, for example, the following expression for $\partial S_x/\partial t$:

$$\partial S_x(t)/dt = \gamma[\mathbf{SH}]_x - \hbar^{-2}\int_0^\infty d\tau\ \mathrm{Sp}\,\hat{S}_x\left[\hat{H}_{\text{dip}}\left[\hat{H}_{\text{dip}}(\tau), \rho_1(t)\right]\right] +$$

$$+i\mu\hbar^{-3}\int_0^\infty d\tau\,\tau\ \mathrm{Sp}\,\hat{S}_x\left[\hat{H}_{\text{dip}}\left[\left[\hat{H}_{\text{dip}}(\tau), \mathbf{H}(t)\hat{\mathbf{S}}\right], \rho_1(t)\right]\right], \qquad (6)$$

where

$$\hat{H}_{\text{dip}}(\tau) = \exp(-i\hbar^{-1}\hat{H}_{\text{ex}}\tau)\,\hat{H}_{\text{dip}}\,\exp(i\hbar^{-1}\hat{H}_{\text{ex}}\tau),$$

and the symbol Sp denotes summation of the diagonal elements. We have omitted the term proportional to $\mathrm{Sp}\,\hat{S}_x[\hat{H}_{\text{dip}}, \rho_1(t)]$, since it takes account of demagnetizing effects [14], which play no role for the spherical specimen under consideration.

Let us consider the second term in (6). Under the sign Sp, we rotate the z-axis into the direction of the instantaneous magnetization. Then Sp takes the form:

$$\sum_{p,p'=-2}^{2} \mathrm{Sp}(\cos\varphi\sin\theta\,\hat{S}_z + \cos\varphi\cos\theta\,\hat{S}_x - \sin\varphi\,\hat{S}_y)\left[\hat{H}^p, \left[\hat{H}^{p'}(\tau), \rho_{10}(t)\right]\right], \quad (7)$$

where φ and θ are the corresponding angles of rotation (in particular, $\cos\varphi\sin\theta = S_x/S$), $\rho_{10}(t) = C\exp[\varkappa(t)\hat{S}_z + \delta(t)\hat{H}_{\text{ex}}]$, and $\hat{H}^p$ is the part of the dipole interaction that causes transitions between levels of the system with a change of the projection of the total spin on the direction of the instantaneous magnetization by p units. It is easy to see that, in the sum over p and p', only the terms with $p = \pm p'$ are nonzero, since otherwise the summation over the spin coordinates contains odd powers of the coordinates, which makes such sums vanish [14]. In addition, the terms with $p = p'$ are equal to zero because in this case Sp vanishes. Thus, in (7) only the terms with $p = -p'$, $p = p' = 0$ remain.

Let us now expand the operators $\hat{H}^p(\tau)$ in Fourier integrals and note that

$$\int_0^\infty d\tau\, e^{-i\omega\tau} = \pi\delta(\omega) - iP\frac{1}{\omega},$$

where P denotes an integral in the sense of the principal value. Then the second term in (6) takes the form $-\lambda(t)S_x(t)/T_1$, where

$$T_1^{-1} = \pi\hbar^{-2}\sum_{p=-2}^{2} p^2 \operatorname{Sp}\rho_{10}\hat{H}_0^{-p}\hat{H}_0^{p}/\operatorname{Sp}\hat{S}_z^2,$$

N is the number of atoms in the lattice,

$$\hat{H}_\omega^p = (2\pi)^{-1}\int_{-\infty}^{\infty} d\tau\, \exp(-i\omega\tau)\,\hat{H}^m(\tau).$$

In the calculations we took into account that

$$[\hat{H}_\omega^p, \rho_{10}(t)] = \rho_{10}(t)\,\hat{H}_\omega^p\left(e^{-x(t)p+\delta(t)\hbar\omega} - 1\right)$$

and that the exponential can be expanded in a series in xp, since for the field values usual in experiment, at $T \gg 1^\circ$K, $xp \ll 1$. (The integrals in the sense of the principal value vanish because of the oddness of the integrand functions.)

As a result of analogous calculations we finally obtain:

$$\partial S_x(t)/dt = \gamma[\mathbf{S}, \mathbf{H}+\Delta\mathbf{H}]_x - \lambda_1(S_x - \chi H_{1x} - \chi H_{px})/T_1 - \lambda_2(T_2^{-1} - T_1^{-1})\left[(S_x S_z H_0 + S_x S_z H_{1z} + S_x^2 H_{1x} + S_x S_y H\right.$$

where $\mathbf{H}_p$ is the demagnetizing field,

$$T_2^{-1} = \pi\hbar^{-2}\sum_{p,p'=-2}^{2}\operatorname{Sp}\rho_{10}\,\hat{H}_0^p[[\hat{H}_0^{p'}\hat{S}_x]\hat{S}_x]/\operatorname{Sp}\hat{S}_z^2,$$

$$\Delta\mathbf{H} = H\hbar^{-2}S^{-1}P\int_{-\infty}^{\infty}\frac{d\omega}{\omega}\frac{d}{d\omega}\left\{\sum_{p=-2}^{2}\operatorname{Sp}\rho_{10}[[\hat{H}_\omega^{-p}\hat{S}_x],[\hat{H}_\omega^p\hat{S}_y]]\right\}.$$

Here $\lambda_1 = \mu H_0 \operatorname{Sp}\hat{S}_z^2/SkT_0$, $\lambda_2 = \mu\operatorname{Sp}\hat{S}_z^2/kT_0$, $\chi = S/H_0$, and T_0 is the temperature. We have taken into account here that $x(t)$ and $\delta(t)$ change only weakly at saturation, as a result of which the dependence of x and δ on t can be neglected (^{2}).

Calculating in the same way $\partial S_z/\partial t$ and $\partial S_y/\partial t$, we finally obtain the equations of motion of the magnetization in the form:

$$\partial\mathbf{S}/\partial t = \gamma[\mathbf{S}, \mathbf{H}+\Delta\mathbf{H}_{\text{eff}}] - \lambda_1(\mathbf{S} - \chi\mathbf{H}_{\text{eff}})/T_1 - \lambda_2(T_2^{-1} - T_1^{-1})[\mathbf{S}[\mathbf{SH}]]/S^2, \quad (8)$$

where $\mathbf{H}_{\text{eff}} = \mathbf{H} + \mathbf{H}_p$.

For $T \gg T_K$, when $\rho_{10} \simeq 1$,

$$T_1^{-1} = \pi\hbar^{-2} \sum_{p,p'=-2}^{2} \mathrm{Sp}[\hat{H}_0^p \hat{S}_z][\hat{H}_0^{p'} \hat{S}_z] / \mathrm{Sp}\, \hat{S}_z^2,$$

$$T_2^{-1} = \pi\hbar^{-2} \sum_{p,p'=-2}^{2} \mathrm{Sp}[\hat{H}_0^p \hat{S}_x][\hat{H}_0^{p'} \hat{S}_x] / \mathrm{Sp}\, \hat{S}_z^2.$$

Let us note that in this case $T_1 = T_2$, since by rotation under the sign Sp these expressions are brought to the same form. Consequently, for $T \gg T_K$ the last term in (8) vanishes and the equations obtained pass into equations of Bloch type ([1,2]).

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Note: Figure translations are in progress. See original paper for figures.