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Abstract

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ON ONE METHOD OF CONVERTING A MULTI-ROW CODE INTO A SINGLE-ROW CODE

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In the case when, in order to specify a certain number C, the numbers $A_1, A_2, \dots, A_m$, satisfying the equality

$$C = \sum_{j=1}^{m} A_j, \tag{1}$$

are used, the set of summands $A_1, A_2, \dots, A_m$ is called an m-row code of the number C.

In digital computing technology, multi-row codes are used mainly to speed up arithmetic operations. In this connection, as a rule, it is necessary to convert the result of an arithmetic operation into a single-row code. Any of the known methods for carrying out such a conversion is based on successively adding each summand of the m-row code to the sum of all preceding ones. This note describes a method that makes it possible to convert a multi-row code into a single-row code considerably faster than when using the methods known up to now.

We consider the case in which the summands A_j $(1 \leqslant j \leqslant m)$ are represented in a positional numeral system with constant weight

$$A_j = \sum_{i=1}^{n} a_{ji} r^{-i}. \tag{2}$$

Here n is the number of digits of A_j; r is the base (an integer) of the numeral system used, with $r \geqslant 2$; a_{ji} is one of the numbers $0, 1, \dots, r-1$.

Substituting (2) into (1) and changing the order of summation, we obtain

$$C = \sum_{i=1}^{n} r^{-i} \sum_{j=1}^{m} a_{ji}. \tag{3}$$

Let the minimum number of digits necessary for representing the digit-wise sum $\sum_{j=1}^{m} a_{ji}$ in the numeral system with base r be denoted by m'. Owing to the integrality of this sum, from the expression

$$\sum_{j=1}^{m} a_{ji} \leqslant m(r-1) \tag{4}$$

it follows that m' is the smallest integer satisfying the inequality

$$m(r-1) \leqslant r^{m'} - 1. \tag{5}$$

Solving it, we obtain:

$$m' =]\log_r(m(r-1)+1)[\tag{6}$$

(the quantity $]d[$ is equal to the smallest integer not less than d). The digit sum $\sum_{j=1}^{m} a_{ji}$ can be represented in the number system with base r in the form

$$\sum_{j=1}^{m} a_{ji} = \sum_{j'=1}^{m'} b_{j'i} r^{j'-1}, \tag{7}$$

where $b_{j'i}$ takes one of the values $0, 1, \ldots, r-1$.

Substituting (7) into (3) and setting $i = i' - (m' - j')$, we obtain

$$C = \sum_{j'=1}^{m'} A'_{j'}, \tag{8}$$

where

$$A'_{j'} = r^{m'-1} \sum_{i'=1}^{n+m'-1} a'_{j'i'} r^{-i'},$$

$$a'_{j'i'} = \begin{cases} b_{j'(i'-(m'-j'))}, & \text{for } 1 \leq i' - (m' - j') \leq n, \\ 0, & \text{in all other cases.} \end{cases}$$

Expression (8) is a representation of the number C in an m'-row code. At the next step the m'-row code of the number C can be transformed into a code with the number of summands equal to $m'' =]\log_r(m'(r-1)+1)[$, and so on. After each transformation step the number of rows in the code will decrease until the next $\widetilde{m}$ is no longer a solution of the inequality

$$\widetilde{m} \leq]\log_r(\widetilde{m}(r-1)+1)[. \tag{9}$$

It can be shown that no integer $\widetilde{m} \geq 3$ is a solution of inequality (9). At the same time $\widetilde{m}_1 = 1$ and $\widetilde{m}_2 = 2$ satisfy the equality

$$\widetilde{m} =]\log_r(\widetilde{m}(r-1)+1)[. \tag{10}$$

Thus the number of rows in the code will decrease until it becomes equal to two. The number of steps s required to transform an m-row code into a two-row code can be computed on the basis of expression (5). The data obtained are given in Table 1.

Table 1

Number of steps s required to transform an m-row code into a two-row code	Maximum value of m, $r=2$	Maximum value of m, $r=3$	Maximum value of m, $r=4$
1	3	4	5
2	7	40	341
3	127	$\frac{1}{2}(3^{40}-1)$	$\frac{1}{3}(4^{341}-1)$
4	$2^{127}-1$	–	–

The block diagram of the device implementing the described method is given in Fig. 1.

To each k-th step of transforming an m-row code into a two-row code there corresponds a group of blocks which performs this step and constitutes the k-th tier of the device. The transformation of the two-row code into a single-row code is carried out by a parallel $(n+m')$-digit adder, which is the $(s+1)$-st tier of the device. The first tier contains n blocks, each of which performs the transformation

$$\sum_{j=1}^{m} a_{ji} \to \sum_{j'=1}^{m'} b_{j'i} r^{j'-1}.$$

Analogous transformations are performed by the blocks of the remaining tiers.

The functional dependences $b_{i'i}$ $(1 \leq i' \leq m')$ on a_{ji} $(1 \leq j \leq m)$ are determined by the chosen radix of the number system. In the most common case, when $r=2$, these dependences have the form

Fig. 1

Figure 1: Fig. 1

$$b_{i'i} = \bigvee_{c_{j'}=1} S_l(a_{1i}, \dots, a_{ji}, \dots, a_{mi}). \quad (11)$$

Here $S_l(a_{1i}, \dots, a_{ji}, \dots, a_{mi})$ denotes the elementary symmetric function of Boolean algebra, equal to 1 if and only if exactly l

Fig. 1

of its arguments are equal to 1; $0 \le l \le m$; $c_{j'}$ is the j'-th digit in the binary notation of the number l:

$$l = \sum_{j'=1}^{m'} c_{j'} 2^{j'-1}. \quad (12)$$

Considerably greater freedom in choosing an optimal circuit for the device is possible with a modification of the method described, which consists in the following. Let m_k be the number of rows in the code obtained after carrying out the k-th step of the transformation (here we take $m_0 = m$). In accordance with the representation of the number m_k in the form

$$m_k = p_k u_k + v_k, \quad (13)$$

where $0 \le v_k < u_k$, the summands of the m_k-row code are divided into groups of u_k summands in each group, except for one group, which may contain a smaller number of summands. Then the u^k-row code of each group is transformed into a code with the number of summands equal to $u'_k = \rceil \log_r(u_k(r-1)+1)\lceil$, while the v_k-row code of the additional group (if such a group exists) is transformed into a code with the number of summands equal to $v'_k = \rceil \log_r(v_k(r-1)+1)\lceil$. As a result of carrying out the $(k+1)$-st transformation step, the number of rows in the code is reduced to the value

$$m_{k+1} = p_k u'_k + v'_k. \quad (14)$$

This operation is repeated until a two-row code is obtained. The latter is transformed into a one-row code by a parallel $(n+m')$-digit adder.

Let us consider the case where $u_0 = \dots = u_k = \dots = u$ (respectively, $u'_0 = \dots = u'_k = \dots = u'$). Using expressions (13) and (14), we estimate from above the number of steps s necessary for transforming an m-row code

into a two-row one:

$$s < \log_{u'} \left(\frac{m}{u+1} \right) + D, \tag{15}$$

where D is a certain constant depending on the chosen r and u. Note that D does not exceed 6 if $u \leqslant 2^{127} - 1$.

It also follows from expressions (13) and (14) that

$$\sum_{k=0}^{s-1} p_k \leqslant \frac{m-2}{u-u'}. \tag{16}$$

Thus, in the given variant, for one digit of the device no more than $\frac{m-2}{u-u'}$ blocks are required that implement a u'-digit sum of u single-digit addends, no more than s less complex blocks, and one single-digit adder.

The proposed method of converting a multi-row code can be used to speed up multiplication. In the case of using a parallel adder based on one of the methods of accelerated addition [1], the described variant with $m = n = 30$–60 (as is usual in computing machines) and $r = 2$, $u = 3$ makes it possible to increase the speed of performing multiplication by a factor of 3-4 in comparison with the known second-order method of accelerating multiplication, and does not require additional equipment for its implementation.

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[1] O. L. MacSorley, Proc. IRE, **49**, No. 1, 67 (1961).

Note: Figure translations are in progress. See original paper for figures.