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Abstract

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ON TAKING INTO ACCOUNT THE RESULTS OF RADIATION MEASUREMENTS FROM SATELLITES IN A NONADIABATIC MODEL OF ATMOSPHERIC MOTIONS

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Radiative heating or cooling, together with heat influxes due to turbulent thermal conductivity and the release of latent heat of condensation, play a rather important role in the development of atmospheric motions. Calculations by D. E. Martin ([6]) have shown that further improvement of weather-forecasting methods undoubtedly requires that nonadiabatic effects be taken into account in prognostic schemes. For this, however, it is necessary to have detailed information on the space-time distribution of heat sources and sinks. In view of the extreme complexity of this problem, it is desirable, first of all, to study the influence of the heat influx, averaged over the entire thickness of the atmosphere, on large-scale motions. Data obtained with the aid of meteorological satellites ([5,7]) make it possible to determine directly the magnitude of this influx.

As the initial system of equations for the problem, let us choose the equation of transport of the velocity vorticity with allowance for the forces of turbulent friction

$$\frac{d_n(\Omega + l)}{dt} - \frac{g^2}{p_0^2}\frac{\partial}{\partial \zeta}\left(K\rho^2\frac{\partial \Omega}{\partial \zeta}\right) = l\frac{\partial \omega}{\partial \zeta} \tag{1}$$

and the equation of heat influx

$$\frac{d_nT}{dt} - \frac{c^2}{R}\frac{\omega}{\zeta} = -\frac{g}{c_p\rho_0}\frac{\partial Q}{\partial \zeta} + \frac{\sigma}{c_p}. \tag{2}$$

Here Ω is the vertical component of the relative velocity vorticity; l is the Coriolis parameter; g is the acceleration of gravity; K is the coefficient of turbulent viscosity; ρ is the density of air; T is the absolute temperature; R is the gas constant; $c^2 = (\gamma_a - \gamma)R^2T/g$; γ and γ_a are the actual and dry-adiabatic vertical temperature gradients; c_p is the specific heat of air at constant pressure; p is pressure; p_0 is the standard pressure at sea level; $\zeta = p/p_0$.

In both equations the symbol d_n/dt denotes the individual time derivative on a horizontal plane, while $\omega = d\zeta/dt$ is the analogue of vertical velocity in the coordinate system x, y, ζ, t.

The first term on the right-hand side of equation (2) characterizes the heat influx to a unit mass of air due to turbulent and radiative heat exchange, and σ/c_p the heat influx due to the latent heat of condensation. If the turbulent heat flux is denoted by D, the upward and downward fluxes of thermal radiation by B and A, respectively, and the ascending and descending fluxes of short-wave radiation by $F^{(1)}$ and $F^{(2)}$, then

$$Q = D + A - B - F^{(2)} - F^{(1)}. \tag{3}$$

As the vertical boundary conditions we shall take

$$\omega = 0 \quad \text{for } \zeta = 0,\ \zeta = 1. \tag{4}$$

To pass in this model from heat inflows to fluxes, let us introduce for consideration a system of equations averaged vertically. Thus, integrating equation (1) with respect to ζ from 0 to 1 and carrying out an approximate allowance for ground friction according to I. A. Kibel' [1], we obtain, under assumption (4),

$$\overline{\frac{d_n(\Omega + l)}{dt}} + K^* \Omega_g = 0. \tag{5}$$

Here

$$K^* \approx \frac{g p_0}{p_0} \sqrt{\frac{K_h l}{2}};$$

K_h is the value of the turbulent-viscosity coefficient at the upper boundary of the surface layer; Ω_g is the geostrophic vorticity at sea level,

$$\overline{(\ )} = \int_0^1 (\ )\, d\zeta.$$

The vertical velocity ω is determined from equation (1):

$$\omega = \frac{1}{l} \left\{ \int_0^\zeta \overline{\frac{d_n(\Omega + l)}{dt}}\, d\zeta - \frac{g^2}{p_0^2} K \rho^2 \frac{\partial \Omega}{\partial \zeta} \right\}. \tag{6}$$

Substituting (6) into (2), averaging the resulting equation over the entire thickness of the atmosphere, and then adding it to equation (6), we obtain

$$\frac{d_n\overline{T}}{dt}+\frac{c^2}{Rl}\left\{\varphi\,\overline{\frac{d_n(\Omega+l)}{dt}}-K^*\Omega_g+\frac{g^2}{p_0^2}\frac{\overline{K_\rho^2}}{\zeta}\frac{\partial\Omega}{\partial\zeta}\right\}=-\frac{g}{c_pp_0}(Q_0-Q_1)+\frac{gLr}{c_pp_0}. \quad (7)$$

Here $\varphi = 1 + \ln\zeta$, L is the latent heat of condensation, and r is the amount of precipitation.

As calculations show, the quantity

$$\frac{g^2}{p_0^2}\frac{\overline{K_\rho^2}}{\zeta}\frac{\partial\Omega}{\partial\zeta}$$

may be neglected in comparison with $K^*\Omega_g$.

We shall now adopt (although this is not obligatory for the present scheme) the condition of thermotropy of the atmosphere ([8]):

$$\nabla T(x,y,\zeta,t) = F(\zeta)\nabla\overline{T}(x,y,t), \quad (8)$$

where ∇ is the gradient operator on the plane. The function $F(\zeta)$ can be determined by statistical processing of observational material. According to L. Berkovsky ([4]), approximately $F(\zeta) = 2\zeta$. Using the statics equation

$$T = -\frac{g\zeta}{R}\frac{\partial H}{\partial\zeta}, \quad (9)$$

it is not difficult, with the aid of (8), to obtain the relations

$$(H)_{\zeta=1} = \overline{H} - \frac{R}{g}\overline{T}, \qquad \nabla H = \nabla\overline{H} + \frac{R\Phi}{g}\nabla\overline{T}, \quad (10)$$

where

$$\Phi = 1 - \int_\zeta^1 \frac{F(\xi)}{\xi}\,d\xi.$$

Let us note that $\overline{F} = 1$ and $\overline{\Phi} = \overline{\varphi} = 0$.

If the motion is regarded as quasigeostrophic, then, on the basis of the preceding, we obtain

$$\overline{\frac{d_nT}{dt}} = \frac{\partial\overline{T}}{\partial t} + \frac{g}{l}(\overline{H},\overline{T});$$

$$\frac{\overline{d_n(\Omega+l)}}{dt}=\frac{g}{l}\left\{\Delta\frac{\partial\overline{H}}{\partial t}+\left(\overline{H},\frac{g}{l}\Delta\overline{H}+l\right)+\frac{R^2\overline{\Phi}^2}{g^2}\left(\overline{T},\frac{g}{l}\Delta\overline{T}\right)\right\}.$$

$$\varphi\frac{\overline{d_n(\Omega+l)}}{dt}=\frac{g}{l}\left\{\frac{R^2\varphi\bar{\Phi}^2}{g^2}\left(\bar{T},\frac{g}{l}\Delta\bar{T}\right)-\frac{R\varphi\bar{\Phi}}{g}\left[\Delta\frac{\partial\bar{T}}{\partial t}\left(\bar{T},\frac{g}{l}\Delta\bar{H}\right)+\left(\bar{H},\frac{g}{l}\Delta\bar{T}\right)+(\bar{T},l)\right]\right\}.$$

The symbols Δ and (a,b) denote, as usual, the Laplace operator on the plane and the Jacobian.

Collecting the results obtained, we finally arrive at the following system of equations of the model:

$$\Delta\frac{\partial\bar{H}}{\partial t}+\left(\bar{H},\frac{g}{l}\Delta\bar{H}+l\right)+\frac{R^2\bar{\Phi}^2}{g^2}\left(\bar{T},\frac{g}{l}\Delta\bar{T}\right)+K^*\left(\Delta\bar{H}-\frac{R}{g}\Delta\bar{T}\right)=0; \quad (11)$$

$$(\Delta-\mu^2)\frac{\partial\bar{T}}{\partial t}-\frac{R\bar{\Phi}^2}{g\varphi\Phi}\left(\bar{T},\frac{g}{l}\Delta\bar{T}\right)+\left(\bar{T},\frac{g}{l}\Delta\bar{H}+l\right)+$$

$$+\left(\bar{H},\frac{g}{l}\Delta\bar{T}\right)-\frac{g\mu^2}{l}(\bar{H},\bar{T})-\frac{gK^*}{R\varphi\Phi}\left(\Delta\bar{H}-\frac{R}{g}\Delta\bar{T}\right)=$$

$$=\frac{\mu^2 g}{c_p p_0}(Q_0-Q_1)+\frac{\mu^2 gLr}{c_p p_0} \quad (12)$$

$$\left(\mu^2=\frac{l^2}{c^2\varphi\Phi}\right).$$

The methods for solving this system of equations are known (see (1)). Let us turn to consideration of the right-hand side of equation (12). The quantity Q_0 represents the radiation balance of the earth' s surface–atmosphere system, which can be calculated directly from satellite measurement data. For prognostic purposes this same quantity is computed from the known formula

$$Q_0=S_0(1-\Gamma)-B_0, \quad (13)$$

where S_0 is the flux of solar radiation arriving at the upper boundary of the atmosphere; Γ is the albedo of the earth' s surface–atmosphere system; B_0 is the outgoing thermal radiation of the Earth and the atmosphere. For Q_1, according to (3), one may obtain the expression

$$Q_1 = (1-\alpha)S - E_{\text{eff}} - D_1. \tag{14}$$

Here α is the albedo of the underlying surface; S is the flux of total radiation; E_{eff} is the effective radiation of the Earth' s surface; D_1 is the turbulent heat flux at the earth' s surface.

The first two terms on the right-hand side of (14), as well as the quantities B_0 and Γ in formula (13), are calculated from vertical atmospheric sounding data, which can in principle be obtained with the aid of meteorological satellites. The calculation of the turbulent heat flux includes the temperature of the underlying surface; for oceanic regions, long-term monthly mean ocean-surface temperatures may be used as this temperature. To compute Q_1 as applied to land conditions, it is advisable to use the heat-balance equation of the underlying surface

$$-D_1 + (1-\alpha)S - E_{\text{eff}} = \Pi - LE, \tag{15}$$

where Π is the heat flux into the soil, and E is the evaporation rate. Then, instead of (14), we obtain:

$$Q_1 = \Pi - LE. \tag{16}$$

If data on the radiative temperature of the soil surface T_{p}, obtained from satellite measurements, are used, then the heat flux into the soil

can be calculated by formula (2)

$$\Pi = \sqrt{\frac{c\rho\chi}{\pi}} \int_{-\infty}^{t} \frac{dT_{\text{p}}}{d\tau} \frac{d\tau}{\sqrt{t-\tau}}. \tag{17}$$

Here c, ρ, and χ are, respectively, the specific heat, density, and coefficient of thermal conductivity of the soil. For winter conditions and in regions where evaporation is small, we obtain $Q_1 \approx \Pi$.

For these same regions the flux Π can be found with the aid of equation (15), if data on soil temperature are lacking. As is known, the turbulent heat flux is a function of the wind speed U and of the temperature difference $T - T_{\text{p}}$ ([3]). In turn, T_{p} is determined through the flux Π in the following way ([2]):

$$T_{\text{p}} = \sqrt{\frac{1}{\pi\chi c\rho}} \int_{-\infty}^{t} \frac{\Pi\, d\tau}{\sqrt{t-\tau}}. \tag{18}$$

Substituting (18) into (15), we obtain an integral equation for determining the heat flux into the soil from the prescribed radiation balance of the underlying surface

$$D_1\left(T-\sqrt{\frac{1}{\pi\chi c\rho}}\int_{-\infty}^{t}\frac{\Pi\,d\tau}{\sqrt{t-\tau}},\,U\right)+\Pi=(1-\alpha)S-E_{\text{eff}}. \tag{19}$$

Finally, for computations of condensation heat inputs at the initial stages, one may use the actual data on the amount of precipitation that has fallen.

The proposed scheme may be used as a forecasting scheme if it includes the moisture-transport equation for precomputing the fields of humidity, cloudiness, and precipitation.

Main Geophysical Observatory
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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.