



---

Soviet-era science, translated into English

# ELASTICITY THEORY

Academician of the Academy of Sciences of the Kirghiz SSR M. Ya.  
LEONOV, P. M. VITVITSKII, S. Ya. YAREMA

1963

SovietRxiv

---

View the original and related papers at <https://sovietrxiv.org/items/ru-196301.33953>

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

Fig. 1

Figure 1: Fig. 1

**Abstract**

**Full Text**

## **ELASTICITY THEORY**

Academician of the Academy of Sciences of the Kirghiz SSR M. Ya. LEONOV,  
P. M. VITVITSKII, S. Ya. YAREMA

### **PLASTICITY BANDS DURING TENSION OF PLATES WITH A CRACK-LIKE CONCENTRATOR**

Plastic deformations in the initial stage of their development are localized in thin layers of material—plasticity bands, also called slip bands <sup>(1,2)</sup>.

The formation and development of such bands was observed by us during tension of thin rectangular plates with a crack-like concentrator.

#### **Fig. 1**

Plates measuring  $200 \times 360$  mm were made from mild sheet steel. The concentrator was introduced in a special device operating on the principle of shears. With its aid, a slit was made along the middle of the plates, usually ending in a small crack continuing from it. To eliminate work hardening and anisotropy, the specimens were normalized with cooling in lattice clamps and then ground to a specified thickness. Before final polishing to a mirror finish of the middle zone of the plates, the latter were subjected to recrystallization annealing.

The specimens were stretched in the direction perpendicular to the concentrator, at low rates ensuring stability of the process. The resulting plasticity bands were observed visually; the sign of the appearance of plastic deformations at a given location was the dulling there of the shiny surface of the plate.

During tension, the following stages of deformation were observed.

1. The first, incubation stage, without visible plastic deformations.
2. The second stage, beginning from the moment when matte spots appear at the crack tips. As the load increases, these spots are elongated in the direction of the concentrator axis, taking the form of narrow wedge-shaped bands that increase in length at an increasing rate. The greatest width of the bands by the end of this stage usually does not exceed 5-6 mm.
3. The beginning of the third stage is marked by the instantaneous appearance of thin plasticity bands inclined at an angle of  $47 \div 54^\circ$  to the axis of the concentrator and extending from its tip over a length of 20-40 mm in

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

the direction of the side edges (Fig. 1). At first 1-2 oblique bands appear, but even with a small increase in load their number grows to 4—two at each end of the concentrator. Further increase in load is accompanied by a rapid increase in the length of the bands, which by the end of the third stage reach the edges of the plate.

**Fig. 2**

4. In the fourth stage, new bands appear simultaneously in many places; under constant load these bands widen and join into zones. The edges of the concentrator diverge and the specimen bulges. The process ends with crack growth under decreasing load.

For an analytical study of the first stages in the development of plastic deformations, we shall assume the material to be ideally elastic-plastic. As the plasticity criterion we take the attainment, by the tangential stresses, of the shear yield limit, which is equal to  $0.5\sigma_t$  ( $\sigma_t$  is the tensile yield limit).

Thanks to the localization of plastic deformation in thin bands, we shall consider the material elastic everywhere except for certain surfaces (lines) on which the displacements undergo discontinuities. In the case under consideration, the discontinuity of displacements in the first two stages of deformation of the plate occurs along a line coinciding with the axis of the concentrator on the segment  $2L$  between the ends of the plasticity bands.

**Fig. 3**

Assuming now the length  $2l$  of the concentrator to be small in comparison with the dimensions of the plate, we may formulate our problem as that of an infinite plate with a rectilinear slit of length  $2L$ , stretched at infinity by stresses  $\sigma_y^\infty = p$  perpendicular to the slit (Fig. 2), under certain conditions on its contour. Since on the axis  $Ox$  the principal stresses are  $\sigma_y(x, 0) > \sigma_x(x, 0) > 0$  ( $|x| > l$ ) and  $\sigma_z = 0$ , the maximum tangential stresses that arise in planes passing through the axis  $Ox$  and inclined at an angle of  $45^\circ$  to the surface of the plate are determined by the equality  $\tau_{\max} = 0.5\sigma_y$ . Consequently, the plasticity condition takes the form  $\sigma_y(x, 0) = \sigma_t$ , and on the contour of the slit it is necessary—

it is necessary to require that

$$\sigma_y(x, \pm 0) = \begin{cases} 0, & |x| < l, \\ \sigma_T, & l \leq |x| \leq L. \end{cases} \quad (1)$$

Fig. 4: graph of  $L/l$  versus  $p_H/\sigma_T$ , with curves labeled 1-4.

Figure 4: Fig. 4: graph of  $L/l$  versus  $p_H/\sigma_T$ , with curves labeled 1-4.

The solution of the problem, found by means of the method of N. I. Muskhelishvili<sup>(3)</sup>, contains the unknown length  $L$ , for whose determination the condition of continuity and boundedness of the stresses at the points  $|x| = L$ ,  $y = 0$  is used. This condition leads to the equality

$$L = l \sec \frac{\pi p}{2\sigma_T}. \quad (2)$$

The stressed state of the plate is determined by the formulas

$$\sigma_y + \sigma_x = p - \frac{2}{\pi} \sigma_T \arg \frac{l^2 - z^2 + li\sqrt{L^2 - l^2} - z\sqrt{z^2 - L^2}}{l^2 - z^2 - li\sqrt{L^2 - l^2} - z\sqrt{z^2 - L^2}}, \quad (3)$$

$$\sigma_y - \sigma_x + 2i\tau_{xy} = p + \frac{2}{\pi} \sigma_T \frac{(z - \bar{z})l\sqrt{L^2 - l^2}}{(z^2 - l^2)\sqrt{z^2 - L^2}}$$

$$(z = x + iy, \quad \bar{z} = x - iy; \quad \sqrt{z^2 - L^2} \rightarrow z \text{ as } |z| \rightarrow \infty).$$

The validity of the obtained solution is violated when the maximum shear stresses on planes perpendicular to the plane of the plate reach the value  $0.5\sigma_T$ , as a result of which inclined bands of plasticity appear. To determine the load at which they arise, we equate  $0.5\sigma_T$  to the maximum values  $\tau_{\max}$  attained at the end of the concentrator ( $|x| = l$ ) on a plane directed toward its axis at the angle

$$\alpha = 0.5 \left[ \pi - \arctg \sqrt{\pi(\pi - 2)} \right] \approx 59^\circ.$$

As a result we find the value of the load corresponding to the beginning of the third stage of deformation:

$$p_* = \sigma_T \sqrt{1 - \frac{2}{\pi}} \approx 0.603 \sigma_T.$$

Fig. 4

The obtained relationships were compared with experimental ones observed on the specimens described earlier. The specimens tested differed from one another in thickness ( $\delta = 1.0; 1.5; 2.0; 2.5$  mm), length of the concentrator ( $2l = 15; 19; 29$  mm), and material (steel grades 08, St. 3, Armco iron). The yield point was defined as the ratio of the load corresponding to the plasticity bands

reaching the edges of the plate to the smallest area of its transverse section. During tension of the specimens, the length of the plastic zones was measured and their shape was recorded. The tests were stopped when the plasticity bands reached the edges of the plate.

The test results were processed in the form of curves expressing the dependence  $L/l = f(p_H/\sigma_T)$  ( $p_H$  is the load referred to the smallest area of the specimen cross section), which were constructed from the test data for 3-4 identical plates.

In Fig. 3 such curves are shown for specimens made of steels 08

( $\sigma_T = 19.8 \text{ kg/mm}^2$ , curves 1-3) and St. 3 ( $\sigma_T = 23.0 \text{ kg/mm}^2$ , curves 4-6) specimens with a concentrator of length  $2l = 18.7 \text{ mm}$  and of different thicknesses (curves 1 and 4— $\delta = 1 \text{ mm}$ , 2 and 5— $\delta = 1.5 \text{ mm}$ , 3 and 6— $\delta = 2.5 \text{ mm}$ ). For curve 1, the experimental points from which it was constructed are shown.

The curves in Fig. 4 correspond to specimens of steel 08, 2.5 mm thick, for three different concentrator lengths (curve 1— $2l = 15 \text{ mm}$ , 2— $2l = 19 \text{ mm}$ , 3— $2l = 29 \text{ mm}$ ). Crosses on these curves indicate the points at which oblique bands appear. Curve 4 is the theoretical dependence given by formula (2).

On the basis of the figures presented and the remaining experimental material, we draw the following conclusions. The theoretical solution reflects fairly well the basic regularity of the development of plastic bands as the load increases. The reasons for the discrepancies between the theoretical and experimental curves should be sought both in the incomplete correspondence of the calculation scheme to the experimental conditions—mainly because of the finite dimensions of the specimens and the finite width of the bands—and in individual factors not taken into account, or not fully taken into account by us (such as plate thickness, strain hardening of the material, concentrator length, and yield strength). Their influence is clearly visible when considering the curves in Figs. 3 and 4.

The deviation of the analytically determined angle of the oblique bands, and of the load at which they appear, from the observed values can be explained (this is confirmed by calculations) by the finite width of the plastic bands and by strain hardening of the already plastically deformed material in the vicinity of the crack tips.

Received  
5 II 1962

## REFERENCES CITED

1. A. Nadai, *Plasticity and Fracture of Solids*, Moscow, 1954.
2. P. F. Koshelev, G. V. Uzhik, *Izv. AN SSSR, Mekhanika i mashinostroenie*, No. 111 (1959).

3. N. I. Muskhelishvili, *Some Basic Problems of the Mathematical Theory of Elasticity*, Moscow, 1954.

*Note: Figure translations are in progress. See original paper for figures.*

*Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.*