Corresponding Member of the USSR Academy of Sciences E. I. GRIGOLYUK,

P. P. CHULKOV

1963

SovietRxiv

Abstract

Full Text

THEORY OF ELASTICITY

Corresponding Member of the USSR Academy of Sciences E. I. GRIGOLYUK, P. P. CHULKOV

ON A GENERAL THEORY OF THREE-LAYER SHELLS WITH LARGE DEFLECTION

In the article [1] a system of nonlinear equations for three-layer shells of nonsymmetric construction was proposed within the framework of the theory of shallow shells, and the displacements of the middle surfaces of the load-bearing layers along the lines of curvature and the deflection were adopted as the basic variables (the compressibility of the package as a whole was not taken into account).

In [2] an analogous system is reduced to three equations in terms of the deflection function, the stress function, and the shear function; in this case a lightweight core was assumed and the intrinsic moments of the load-bearing layers were not taken into account.

It is shown below that the general nonlinear equilibrium equations of shallow three-layer shells of nonsymmetric construction with a rigid core and with allowance for the intrinsic moments of the load-bearing layers can be represented in the form of two equations in terms of the force function and the displacement function; their structure is in many respects analogous to the structure of the corresponding equations of single-layer shallow shells.

We formulate the assumptions and hypotheses on which the results of the work are based. The total thickness of the package of the shell wall is constant and negligibly small in comparison with its other dimensions. The load-bearing layers have constant, but different, thicknesses and are made of different isotropic materials; for them the Kirchhoff–Love hypotheses are established. The material of the core is transversely isotropic. Transverse compression of the core is not taken into account. The tangential displacements of points of the core are approximated by linear functions of the transverse coordinate. The strains are assumed elastic.

1. As the reference surface we take the middle surface of the core, referred to orthogonal coordinates x_i $(i = 1, 2)$. The normal coordinate $z > 0$ if it is measured in the direction of the outward normal, i.e., toward the first load-bearing layer.

Let k_{ij} $(i, j = 1, 2)$ denote the curvatures and torsion of the coordinate lines; h_1, h_2, $2c$ the thicknesses of the first and second load-bearing layers and of the core, respectively; E_k and ν_k $(k = 1, 2, 3)$ the Young' s moduli and Poisson' s ratios of the materials of the load-bearing layers and the core; G the shear modulus of the core material for the planes $x_i z$; w the normal displacements of points of the shell; u_i $(i = 1, 2)$ the tangential displacements of points of the reference surface; $2a_i$ $(i = 1, 2)$ the absolute shear of the bonding planes of the load-bearing layers and the core; δ_{ij} the Kronecker symbol.

On the basis of the adopted hypotheses, the tangential displacements of points of the package are written as follows:

$$u_i^z = \begin{cases} u_i + a_i - (z - c)\dfrac{\partial w}{\partial x_i}, & c \le z \le c + h_1, \\ u_i + \dfrac{z}{c} a_i, & -c \le z \le c, \\ u_i - a_i - (z + c)\dfrac{\partial w}{\partial x_i}, & -h_2 - c \le z \le -c. \end{cases} \tag{1}$$

The longitudinal strains and shear angles will be:

for the load-bearing layers

$$e_{ij}^k = \varepsilon_{ij} \mp \alpha_{ij} + (z \mp c)\chi_{ij} \qquad (k = 1, 2;\ i, j = 1, 2); \tag{2}$$

for the core

$$e_{ij}^3 = \varepsilon_{ij} + \frac{z}{c}\alpha_{ij}, \qquad e_{i3}^3 = \frac{a_i}{c} + \frac{\partial w}{\partial x_i} \qquad (i, j = 1, 2), \tag{3}$$

where

$$\varepsilon_{ij} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + 2k_{ij} w + \frac{\partial w}{\partial x_i}\frac{\partial w}{\partial x_j}\right]. \tag{4}$$

$$\alpha_{ij} = \frac{1}{2}\left[\frac{\partial a_i}{\partial x_j} + \frac{\partial a_j}{\partial x_i}\right], \qquad \chi_{ij} = -\frac{\partial^2 w}{\partial x_i \partial x_j}.$$

According to Hooke' s law, the stresses are equal to:

in the load-bearing layers

$$\sigma_{ij}^k = \frac{E_k}{1 - \nu_k^2}\left[(1 - \nu_k) e_{ij}^k + \nu_k e^k \delta_{ij}\right], \qquad e^k = e_{11}^k + e_{22}^k \qquad (k = 1, 2); \tag{5}$$

in the core

$$\sigma_{ij}^3 = \frac{E_3}{1-\nu_3^2}\left[(1-\nu_3)e_{ij}^3 + \nu_3 e^3\delta_{ij}\right], \qquad \sigma_{i3}^3 = G\left(\frac{a_i}{c} + \frac{\partial w}{\partial x_i}\right); \qquad e^3 = e_{11}^3 + e_{22}^3. \tag{6}$$

The equilibrium equations of a shell subjected to an external normal pressure q have the form ([1]):

$$\frac{\partial T_{1i}}{\partial x_1} + \frac{\partial T_{i2}}{\partial x_2} = 0, \qquad \frac{\partial H_{i1}}{\partial x_1} + \frac{\partial H_{i2}}{\partial x_2} = Q_i^3 \qquad (i=1,2), \tag{7}$$

$$\frac{\partial^2 M_{11}}{\partial x_1^2} + 2\frac{\partial^2 M_{12}}{\partial x_1 \partial x_2} + \frac{\partial^2 M_{22}}{\partial x_2^2} + \frac{\partial Q_1^3}{\partial x_1} + \frac{\partial Q_2^3}{\partial x_2} - \sum_{i,j=1}^{2} T_{ij}(k_{ij} + \chi_{ij}) - q = 0,$$

where

$$T_{ij} = \int_c^{c+h_1} \sigma_{ij}^1\,dz + \int_{-c}^{c} \sigma_{ij}^3\,dz + \int_{-c-h_2}^{-c} \sigma_{ij}^2\,dz, \qquad Q_i^3 = \int_{-c}^{c} \sigma_{i3}^3\,dz,$$

$$H_{ij} = \int_{-c}^{c} \sigma_{ij}^3 z\,dz + c\left[\int_c^{c+h_1} \sigma_{ij}^1\,dz - \int_{-c-h_2}^{-c} \sigma_{ij}^2\,dz\right], \tag{8}$$

$$M_{ij} = \int_c^{c+h_1} \sigma_{ij}^1 (z-c)\,dz + \int_{-c-h_2}^{-c} \sigma_{ij}^2 (z+c)\,dz.$$

The first four equations of system (7), within the accuracy of the initial assumptions, are identically satisfied by introducing the stress function F and the displacement function χ according to the formulas

$$T_{ij} = \left(\delta_{ij}\Delta - \frac{\partial^2}{\partial x_i \partial x_j}\right)F, \qquad w = \left(1 - \frac{c^2\omega_1}{\xi}\Delta\right)\chi,$$

$$a_i = -\frac{\partial}{\partial x_i}\left(1 + \frac{c^2\omega_2}{\xi}\Delta\right)c\chi. \tag{9}$$

Expressing ε_{ij} in terms of F and χ, and using the strain compatibility equation, we have

$$\Delta\Delta F - B\left(\mu_1 - \frac{c^2\omega_3}{\beta}\Delta\right)\Delta\Delta c\chi =$$

$$= B(1-b^2)\left[\frac{\partial^2}{\partial x_1^2}(k_{22}w) - 2\frac{\partial^2}{\partial x_1 \partial x_2}(k_{12}w) + \frac{\partial^2}{\partial x_2^2}(k_{11}w) - \frac{\partial^2 w}{\partial x_1^2}\frac{\partial^2 w}{\partial x_2^2} + \left(\frac{\partial^2 w}{\partial x_1 \partial x_2}\right)^2\right]. \tag{10}$$

The fifth equation of the system (7) gives

$$Bc^2\left(\frac{c^2\omega}{\beta}\Delta - \mu\right)\Delta\Delta\chi - \frac{\partial^2 F}{\partial x_2^2}(k_{11} + \chi_{11}) +$$

$$+2\frac{\partial^2 F}{\partial x_1 \partial x_2}(k_{12} + \chi_{12}) - \frac{\partial^2 F}{\partial x_1^2}(k_{22} + \chi_{22}) - q = 0. \tag{11}$$

Here

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}, \qquad B = \frac{E_1 h_1}{1-\nu_1^2} + \frac{E_2 h_2}{1-\nu_2^2} + \frac{2E_3 c}{1-\nu_3^2},$$

$$\lambda_1 = \frac{E_1 h_1}{B(1-\nu_1^2)}, \qquad \lambda_2 = \frac{E_2 h_2}{B(1-\nu_2^2)}, \qquad \lambda_3 = \frac{2E_3 c}{B(1-\nu_3^2)},$$

$$\beta = \frac{2Gc}{B}, \qquad t_1 = \frac{h_1}{2c}, \qquad t_2 = \frac{h_2}{2c}, \qquad b = \lambda_1\nu_1 + \lambda_2\nu_2 + \lambda_3\nu_3,$$

$$\omega_1 = 4\lambda_1\lambda_2 + \lambda_3\left(\frac{4}{3} - \lambda_3\right), \qquad \omega_2 = \lambda_3(\lambda_1 t_1 + \lambda_2 t_2) + 2\lambda_1\lambda_2(t_1 + t_2),$$

$$\omega_3 = \frac{1}{3}\lambda_3^2[\lambda_1 t_1(\nu_1 - \nu_3) + \lambda_2 t_2(\nu_3 - \nu_2)] +$$

$$+\lambda_1\lambda_2\lambda_3\left[(\nu_1 - \nu_3)\left(\frac{4}{3}t_1 - \frac{2}{3}t_2\right) + (\nu_3 - \nu_2)\left(\frac{4}{3}t_2 - \frac{2}{3}t_1\right)\right],$$

$$\omega = \frac{4}{3}\left(\lambda_1\lambda_2 + \frac{\lambda_3^2}{3}\right)(\lambda_1 t_1^2 + \lambda_2 t_2^2) +$$

$$+\frac{4}{9}\lambda_3\left[(\lambda_1 t_1 - \lambda_2 t_2)^2 + 4\lambda_1\lambda_2\left(t_1^2 - \frac{t_1 t_2}{4} + t_2^2\right)\right], \tag{12}$$

$$\mu_1 = \lambda_1\lambda_2(2 + t_1 + t_2)(\nu_2 - \nu_1) + \lambda_1\lambda_3(1 + t_2)(\nu_3 - \nu_1) +$$

$$+\lambda_2\lambda_3(1+t_2)(\nu_2-\nu_3),$$

$$\mu = 4\lambda_1\lambda_2(1+t_1+t_2)+2\lambda_3\left(\frac{2}{3}-\frac{1}{2}\lambda_3+\lambda_1 t_1+\lambda_2 t_2\right)+$$

$$+\frac{4}{3}(\lambda_1 t_1^2+\lambda_2 t_2^2)-(\lambda_1 t_1-\lambda_2 t_2)^2.$$

If the thickness of the core greatly exceeds the thickness of each of the load-bearing layers, then the intrinsic moments of the load-bearing layers may be neglected; in this case equation (11) takes the form

$$Bc^2(\omega_1+\omega_2)\Delta\Delta\chi+\frac{\partial^2 F}{\partial x_2^2}(k_{11}+\chi_{11})-2\frac{\partial^2 F}{\partial x_1\partial x_2}(k_{12}+\chi_{12})+$$

$$+\frac{\partial^2 F}{\partial x_1^2}(k_{22}+\chi_{22})+q=0. \qquad (13)$$

In addition, we note that the term

$$B\left(\mu_1-\Delta\frac{c^2\omega_3}{\beta}\right)\Delta\Delta c\chi,$$

appearing in equation (10), is in many cases negligibly small.

2. Equations (10)–(11), without any additions, are also suitable for not-too-shallow shells of zero Gaussian curvature. For a shell with Gaussian curvature different from zero, these equations are applicable if its stress state is characterized by the influence of the edge effect.

Institute of Hydrodynamics
Siberian Branch of the Academy of Sciences of the USSR

Received
19 I 1963

References

1. T. N. Vashchina, *Izv. AN SSSR, Mekhanika i mashinostroenie*, No. 5 (1961).

2. Kh. M. Mushtari, *Izv. AN SSSR, Mekhanika i mashinostroenie*, No. 2, 24 (1961).