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Abstract

Full Text

MATHEMATICS

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ON A CLASS OF ALGORITHMS OVER FINITE SETS

(Presented by Academician S. L. Sobolev on 28 II 1963)

In the paper ⁽¹⁾ algorithms for simplifying disjunctive normal forms (d.n.f.) were described. The algorithm was considered in ⁽¹⁾ as a process of successive accumulation of information about the conjunctions entering into the d.n.f. A substantial shortcoming of ⁽¹⁾ is the circumstance that, for each conjunction, only the fact of the conjunction's membership or nonmembership in $\mathfrak{A}_{\Sigma M}$ or $\mathfrak{A}_{\cap M}$ was remembered. Only this information, if it could be obtained in the course of the algorithm's operation, was used at later stages of the algorithm. It is natural to describe and study algorithms with a richer memory. At the same time it turns out that the schemes considered in ⁽¹⁾ fit the description of a broad class of algorithms computing information about elements of finite sets.

I. Let a family $\{\mathfrak{M}\}$ of finite sets be given. For each pair $(\mathfrak{A}, \mathfrak{M})$ (here $\mathfrak{M} \in \{\mathfrak{M}\}$, $\mathfrak{A} \in \mathfrak{M}$) a system of two-place predicates $P_1(\mathfrak{A}, \mathfrak{M}), \dots, P_k(\mathfrak{A}, \mathfrak{M})$ is given. For example, if: 1) $\{\mathfrak{M}\}$ is the set of all shortened normal forms \mathfrak{R} of functions of Boolean algebra in variables entering into the set x_1, \dots, x_n, \dots ; 2) \mathfrak{R} is considered as the set of elementary conjunctions \mathfrak{A}_i entering into \mathfrak{R} , then examples of predicates are: $P_1(\mathfrak{A}, \mathfrak{R})$ —the conjunction \mathfrak{A} is contained in at least one of the minimal d.n.f.'s of the function realized by \mathfrak{R} ; $P_2(\mathfrak{A}, \mathfrak{R})$ —the conjunction \mathfrak{A} is contained in all irredundant d.n.f.'s of the function realized by \mathfrak{R} .

II. We shall assign to elements of sets from the system $\{\mathfrak{M}\}$ tuples of marks $(\alpha_1 \dots \alpha_n)$ and consider elements with marks (notation: $\mathfrak{A}^{\alpha_1 \dots \alpha_k}$). The tuple $(\alpha_1 \dots \alpha_k)$ will be called the **mark vector of the element** \mathfrak{A} in the set \mathfrak{M} , and α_i the i -th **coordinate of the vector**. Each of the coordinates of the vector $(\alpha_1 \dots \alpha_k)$ may take values from the set $\{\Delta, 0, 1\}$. A mark vector is called **admissible for the pair** $(\mathfrak{A}, \mathfrak{M})$, $\mathfrak{A} \in \mathfrak{M}$, if the following condition is fulfilled: for all coordinates α_j of the vector $(\alpha_1 \dots \alpha_k)$ taking the values 0 or 1, the relation $\alpha_j = P_j(\mathfrak{A}, \mathfrak{M})$ holds. We shall consider sets from $\{\mathfrak{M}\}$ composed of elements with admissible marks. Sets consisting of identical elements with different marks will be regarded as different.

III. To each pair $(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M})$, $\mathfrak{A}^{\alpha_1 \dots \alpha_k} \in \mathfrak{M}$, we associate a set $S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M})$ of elements of \mathfrak{M} , which we shall call the **neighborhood** of $\mathfrak{A}^{\alpha_1 \dots \alpha_k}$ in \mathfrak{M} with center $\mathfrak{A}^{\alpha_1 \dots \alpha_k}$, if the following conditions are fulfilled:

1°. $\mathfrak{A}^{\alpha_1 \dots \alpha_k} \in S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M})$.

2°. Denote by $M[S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M})]$ the set of elements without marks entering into $S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M})$, and by $M(\mathfrak{M})$ the set of elements without marks entering into \mathfrak{M} . Let $\mathfrak{A}^{\alpha_1 \dots \alpha_k} \in \mathfrak{M}_1$, $\mathfrak{A}^{\beta_1 \dots \beta_k} \in \mathfrak{M}_2$, $M[S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M}_1)] \subseteq M(\mathfrak{M}_2) \subseteq M(\mathfrak{M}_1)$. Then

$$M[S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M}_1)] = M[S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M}_2)].$$

From 2° it easily follows that if \mathfrak{M}_1 and \mathfrak{M}_2 are composed of identical elements with different marks, $\mathfrak{A}^{\alpha_1 \dots \alpha_k} \in \mathfrak{M}_1$ and $\mathfrak{A}^{\beta_1 \dots \beta_k} \in \mathfrak{M}_2$, then

$$M[S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M}_1)] = M[S(\mathfrak{A}^{\beta_1 \dots \beta_k}, \mathfrak{M}_2)].$$

IV. The system of functions $\varphi_1, \dots, \varphi_k$

$$\varphi_i = \varphi_i[\mathfrak{A}^{\alpha_1 \dots \alpha_k}, S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M})] = (\beta_1 \dots \beta_k), \quad \mathfrak{A}^{\alpha_1 \dots \alpha_k} \in \mathfrak{M},$$

is defined on all pairs $(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M}))$ such that $\mathfrak{A}^{\alpha_1 \dots \alpha_k} \in \mathfrak{M}$ ($i = 1, 2, \dots, k$). The values φ_i satisfy the relation

$$\varphi_i[\mathfrak{A}^{\alpha_1 \dots \alpha_{i-1} \alpha_i \alpha_{i+1} \dots \alpha_k}, S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M})] = (\alpha_1 \dots \alpha_{i-1} \beta_i \alpha_{i+1} \dots \alpha_k).$$

Let \mathfrak{M} be a set composed of elements with admissible labels, $\mathfrak{A}^{\alpha_1 \dots \alpha_k} \in \mathfrak{M}$, and

$$\varphi_j[\mathfrak{A}^{\alpha_1 \dots \alpha_k}, S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M})] = (\beta_1 \dots \beta_k).$$

The function φ_j is called admissible if the set obtained by replacing in \mathfrak{M} the element $\mathfrak{A}^{\alpha_1 \dots \alpha_k}$ by $\mathfrak{A}^{\beta_1 \dots \beta_k}$ is composed of elements with admissible labels.

V. Let us introduce a partial ordering in certain sets.

1°. $M_1 = \{\Delta, 0, 1\}$, $\Delta < 0$, $\Delta < 1$.

2°. M_2 is the set of vectors of dimension k whose coordinates take values in M_1 :

$$(\alpha_1 \dots \alpha_k) \leq (\beta_1 \dots \beta_k), \quad \text{if } \alpha_i \leq \beta_i, \quad i = 1, 2, \dots, k.$$

3°. M_3 is the set of elements with labels: $\mathfrak{A}^{\alpha_1 \dots \alpha_k} \leq \mathfrak{A}^{\beta_1 \dots \beta_k}$, if $(\alpha_1 \dots \alpha_k) \leq (\beta_1 \dots \beta_k)$. If $\mathfrak{A}^{\alpha_1 \dots \alpha_k} \leq \mathfrak{A}^{\beta_1 \dots \beta_k}$ and $\mathfrak{A}^{\beta_1 \dots \beta_k} \leq \mathfrak{A}^{\alpha_1 \dots \alpha_k}$, then the elements $\mathfrak{A}^{\alpha_1 \dots \alpha_k}$ and $\mathfrak{A}^{\beta_1 \dots \beta_k}$ will be called equal with respect to information. Notation: $\mathfrak{A}^{\alpha_1 \dots \alpha_k} \cong \mathfrak{A}^{\beta_1 \dots \beta_k}$.

4°. M_4 is the set of sets of elements with labels. $\mathfrak{M}_1 \leq \mathfrak{M}_2$, if $M(\mathfrak{M}_1) = M(\mathfrak{M}_2)$ and from the conditions $\mathfrak{A}^{\alpha_1 \dots \alpha_k} \in \mathfrak{M}_1$, $\mathfrak{A}^{\beta_1 \dots \beta_k} \in \mathfrak{M}_2$ it follows that $(\alpha_1 \dots \alpha_k) \leq (\beta_1 \dots \beta_k)$. If $\mathfrak{M}_1 \leq \mathfrak{M}_2$ and $\mathfrak{M}_2 \leq \mathfrak{M}_1$, then \mathfrak{M}_1 and \mathfrak{M}_2 will be called equal with respect to information. Notation: $\mathfrak{M}_1 \cong \mathfrak{M}_2$.

5°. M_5 is the set of neighborhoods $(S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M}))$. $S = S^1(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M}_1) \leq S(\tilde{\mathfrak{A}}^{\beta_1 \dots \beta_k}, \mathfrak{M}_2) = S^2$, if $\mathfrak{A}^{\alpha_1 \dots \alpha_k} \leq \tilde{\mathfrak{A}}^{\beta_1 \dots \beta_k}$, $M[S^1] = M[S^2]$, and for any elements $\mathfrak{A}_i^{\gamma_1 \dots \gamma_k} \in S^1$, $\mathfrak{A}_i^{\delta_1 \dots \delta_k} \in S^2$ the relation $(\gamma_1 \dots \gamma_k) \leq (\delta_1 \dots \delta_k)$ holds. If $S^1 \leq S^2$ and $S^2 \leq S^1$, then the neighborhoods S^1 and S^2 will be called equal with respect to information. Notation: $S^1 \cong S^2$.

VI. The function $\varphi_j[\mathfrak{A}^{\alpha_1 \dots \alpha_k}, S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M})]$ is called monotone if from the relation

$$S_1 = S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M}_1) \leq S(\tilde{\mathfrak{A}}^{\beta_1 \dots \beta_k}, \mathfrak{M}_2) = S_2$$

it follows that

$$\varphi_j[\mathfrak{A}^{\alpha_1 \dots \alpha_k}, S_1] \leq \varphi_j[\tilde{\mathfrak{A}}^{\beta_1 \dots \beta_k}, S_2].$$

In what follows we shall consider only monotone functions φ_j .

VII. Algorithms A_π . Let M be an arbitrary finite set composed of elements with labels; $N = \{1, 2, \dots, k\}$. Consider the set $M \tilde{\times} N$ of all pairs $\{\mathfrak{A}^{\alpha_1 \dots \alpha_k}, j\}$, where $\mathfrak{A}^{\alpha_1 \dots \alpha_k} \in M$, $j \in N$, such that $\alpha_j = \Delta$. The algorithm A_π completely orders the set $M \tilde{\times} N$. The algorithms A_π^1 and A_π^2 are called distinct if they order at least one set $M \tilde{\times} N$ differently.

VIII. Δ is the operator with respect to the system $i_1 \dots i_l$ on the set \mathfrak{M} . We replace in the label vectors of all elements of \mathfrak{M} the values of all coordinates with numbers different from i_1, \dots, i_l by Δ . Notation: $\Delta_{i_1 \dots i_l}(\mathfrak{M})$.

IX. A partition of the system of predicates P_1, \dots, P_k into two sets is given. The elements P_{i_1}, \dots, P_{i_l} of the first set will be called basic predicates.

X. Definition of an algorithm A over a finite set \mathfrak{M} . The algorithm is completely determined by the system of predicates P_1, \dots, P_k , by the selection of the basic predicates P_{i_1}, \dots, P_{i_l} , by the system of functions $\varphi_1, \dots, \varphi_k$, and by the algorithm A_π .

Let $\mathfrak{M} = \{\mathfrak{A}_i^{\alpha_{i_1} \dots \alpha_{i_k}}\}$, $i = 1, 2, \dots, m$.

First stage of the algorithm. The algorithm A_π is applied to the set $M \times N$. Here $M = M(\mathfrak{M})$, $N = \{1, 2, \dots, k\}$. The first pair in order $[\mathfrak{A}^{\alpha_1 \dots \alpha_k}, j]$ is selected. The function is computed

$$\varphi_j[\mathfrak{A}^{\alpha_1 \dots \alpha_k}, S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M})] = (\beta_1 \dots \beta_k).$$

The element $\mathfrak{A}^{\alpha_1 \dots \alpha_k}$ is replaced by $\mathfrak{A}^{\beta_1 \dots \beta_k}$. If $(\alpha_1 \dots \alpha_k) = (\beta_1 \dots \beta_k)$, the second pair in order is taken from $M \times N$, and so on. If for all elements $(\mathfrak{A}^{\gamma_1 \dots \gamma_k}, j) \in M \times N$ the relation $\varphi_j[\mathfrak{A}^{\gamma_1 \dots \gamma_k}, S(\mathfrak{A}^{\gamma_1 \dots \gamma_k}, \mathfrak{M})] = (\gamma_1, \dots, \gamma_k)$ holds, the algorithm A terminates after all elements of $M \times N$ have been examined. Otherwise, the first pair $(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, j)$ is found for which

$$\varphi_j[\mathfrak{A}^{\alpha_1 \dots \alpha_k}, S(\mathfrak{A}^{\alpha_1 \dots \alpha_k}, \mathfrak{M})] = (\beta_1 \dots \beta_k) \neq (\alpha_1 \dots \alpha_k).$$

The element $\mathfrak{A}^{\alpha_1 \dots \alpha_k}$ is replaced by $\mathfrak{A}^{\beta_1 \dots \beta_k}$. The set \mathfrak{M} passes into \mathfrak{M}_1 . It is checked whether in \mathfrak{M}_1 there remain elements in whose label vector at least one coordinate with number from the set i_1, \dots, i_l is equal to Δ . If there are no such elements, the algorithm terminates. Otherwise only the first stage of the algorithm terminates. Suppose n stages of the algorithm A have been performed and the set \mathfrak{M} has passed into \mathfrak{M}_n . The description of the $(n + 1)$ -st stage exactly repeats the description of the first stage, if instead of the set $\mathfrak{M} \subseteq \mathfrak{M}_0$ one considers the set \mathfrak{M}_n . Obviously, by the monotonicity and admissibility of φ_j , $j = 1, 2, \dots, k$, the algorithm terminates after a finite number of stages.

To each algorithm A and set $\mathfrak{M} \subseteq \mathfrak{M}_0$ one can associate a sequence $\mathfrak{M}_0, \mathfrak{M}_1, \dots, \mathfrak{M}_\alpha$ (notation: $\pi(\mathfrak{M}, A)$) possessing the following properties:

1⁰. The algorithm A successively transforms \mathfrak{M}_0 into $\mathfrak{M}_1, \dots, \mathfrak{M}_{i-1}$ into $\mathfrak{M}_i, \dots, \mathfrak{M}_{\alpha-1}$ into \mathfrak{M}_α .

2⁰. Adjacent elements of the sequence differ only by the label vectors of a single element, and these vectors have only one differing coordinate. The set \mathfrak{M}_α will be called the **image** of \mathfrak{M} in the algorithm A . To each algorithm A one can put into one-to-one correspondence the collection

$$(P_1, \dots, P_k, P_{i_1}, \dots, P_{i_l}, \varphi_1, \dots, \varphi_k, A_\pi).$$

We shall say that the set of algorithms $\{A\}$ belongs to the class

$$K(P_1 \dots P_k, P_{i_1} \dots P_{i_l}, \varphi_1 \dots \varphi_k),$$

if there are associated with them collections differing only in the last elements (the algorithms A_π). Let A and B be algorithms from the class

$$K(P_1 \dots P_k, P_{i_1} \dots P_{i_l}, \varphi_1 \dots \varphi_k).$$

Definition. Algorithms A and B will be called **equivalent with respect to the system** P_{i_1}, \dots, P_{i_l} , if for any set \mathfrak{M} , consisting of elements with admissible labels, such that $M(\mathfrak{M}) \in \{\mathfrak{M}\}$, we have

$$\Delta_{i_1 \dots i_l}(A(\mathfrak{M})) \subseteq \Delta_{i_1 \dots i_l}(B(\mathfrak{M})).$$

Theorem. If $\varphi_1, \dots, \varphi_k$ are monotone functions, then all algorithms from the class

$$K(P_1 \dots P_k, P_{i_1} \dots P_{i_l}, \varphi_1 \dots \varphi_k)$$

are equivalent with respect to the system

$$P_{i_1}, \dots, P_{i_l}.$$

Let

$$\Delta_{i_1 \dots i_l}(A(\mathfrak{M})) = \bigcup_{i=1}^m \mathfrak{A}_i^{\alpha_{i_1} \dots \alpha_{i_l}}.$$

Definition. The set

$$J = \bigcup_{i=1}^m (\alpha_{i_1}, \dots, \alpha_{i_l}^i)$$

will be called the **local information over \mathfrak{M} in the class $K(P_1 \dots P_k, P_{i_1} \dots P_{i_l}, \varphi_1 \dots \varphi_k)$.**

By virtue of the theorem formulated above, the set J is determined uniquely.

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REFERENCES

1. Yu. I. Zhuravlev, Collection *Problems of Cybernetics*, No. 8, 1962, p. 5.

Note: Figure translations are in progress. See original paper for figures.

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