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Abstract

Full Text

HYDROMECHANICS

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DETERMINATION OF THE SHAPE AND PARAMETERS OF THE FRONT OF A SHOCK WAVE IN AN EXPLOSION IN AN INHOMOGENEOUS MEDIUM

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The present note indicates an approximate method for calculating the parameters of an explosion wave, as well as the shape of the wave in the case of a point explosion in an inhomogeneous medium.

1. Let us consider an explosion in a quiescent ideal medium whose initial density and pressure vary according to the laws

$$\rho_1 = \rho_0 \Omega_1(z/H), \quad p_1 = p_0 \Omega_2(z/H); \quad (1)$$

here z is the coordinate, H is a constant with the dimension of length; ρ_0 is the density at $z = 0$; p_0 is the pressure at $z = 0$.

We shall assume that the explosion of a spherical charge of radius r_0 occurred at $z = 0$ at time $t = 0$. We denote the energy of the explosion by E_0 and suppose that the energy E_0 was released instantaneously. For the case of a perfect gas the determining parameters of the problem are

$$r, \theta, t, E_0, \rho_0, p_0, H, r_0, \gamma, \quad (2)$$

where r is the length of the radius vector of the spherical coordinate system; θ is the angle measured from the z -axis ($z = r \cos \theta$); γ is the adiabatic exponent. From the parameters (2) one can form the following dimensionless combinations:

$$R = r/r^0, \quad \theta, \quad \tau = t/t^0, \quad h = H/r^0, \quad \delta = r_0/r^0, \quad \gamma, \quad (3)$$

where $r^0 = (E_0/p_0)^{1/3}$ is the dynamic length, $t^0 = r^0(\rho_0/p_0)^{1/2}$ is the dynamic time.

For a point explosion ($r_0 = 0$) the parameter δ drops out; for an explosion in a homogeneous medium ($\rho_1 = \text{const}$; $p_1 = \text{const}$) the parameter h drops

out. Using the conclusions of dimensional theory, we obtain that the dimensionless quantities sought, for example the pressure p/p_0 , will depend on the dimensionless combinations indicated in (3):

$$p/p_0 = P(R, \tau, \theta, h, \delta, \gamma). \quad (4)$$

It follows from (4) that a calculation made for certain fixed E_0, p_0, ρ_0 cannot be used for conversion to other values of these parameters without changing H and r_0 . Thus, the calculation of problems on an explosion in an inhomogeneous atmosphere is greatly complicated, since it is necessary to carry out computations for a series of parameters h, δ, γ . The solutions of the problems become still more complicated if the medium is not a perfect gas with constant heat capacities (for example, water, air with allowance for dissociation and ionization). In the case of ideal two-parameter media different from a perfect gas, constants $\tilde{p}_i, \tilde{\rho}_i$ ($i = 1, 2, \dots, n$), entering into the expression for the internal energy of the gas and having the dimensions of pressure and density, respectively, are added to the parameters (2).

To the system of parameters (3) there will be added $\tilde{p}_i/p_0 = \gamma_i, \tilde{\rho}_i/\rho_0 = \beta_i$, which will vary with changes in p_0, ρ_0 .

Since the calculation of even a single variant of the explosion problem is rather laborious, the circumstance noted leads to the necessity of finding various approximate methods for determining the parameters of explosion waves. Analytical methods for solving the nonstationary equations of gas dynamics have at present been developed for limiting cases of sufficiently strong⁽¹⁾ or sufficiently weak shock waves^(2,3). Of practical interest, however, is the development of an explosion from its strong initial stage to the stage of degeneration into sound waves. Let us note that

In [2] one of the approximate methods is given for determining the pressure at large distances in an explosion in an inhomogeneous atmosphere, based on the use of asymptotic laws of attenuation of shock waves. Below we shall give some approximate methods for determining the parameters of blast waves, based on the results of solving equations and problems of gas dynamics for strong and weak shock waves, or on experimental data on the distribution of some characteristic of the wave front.

The conditions at the shock wave may be written as follows:

$$\rho_1 D = \rho_2 (D - v_2) = j, \quad v_2 j = p_2 - p_1, \quad \varepsilon_2 - \varepsilon_1 = \frac{p_2 + p_1}{2} \left(\frac{1}{\rho_1} - \frac{1}{\rho_2} \right), \quad (5)$$

where ε is the internal energy; v_2 is the velocity behind the discontinuity, directed along the normal to the shock-wave surface. The subscript 2 refers to quantities behind the wave front.

If, for a fixed medium, one of the quantities p_2, ρ, v_2, D is prescribed as a function of r_2, θ , then all the others of these quantities will be found from relations (5).

Let us consider the question of determining the shape of the wave and the law of its variation with time. Introduce the dimensionless quantity $W = D/a_1$, where a_1 is the speed of sound in the undisturbed medium.

As is known, if $r_2 = r_2(t, \theta)$ is the law of variation of the shock-wave front, then

$$D = \frac{\partial r_2}{\partial t} \left[1 + \left(\frac{1}{r^2} \frac{\partial r_2}{\partial \theta} \right)^2 \right]^{-1/2}.$$

Passing to dimensionless variables, we find

$$\beta \frac{\partial l}{\partial \tau} = W(l, \theta) \left[\left(\frac{1}{l} \frac{\partial l}{\partial \theta} \right)^2 + 1 \right]^{1/2}, \quad (6)$$

where

$$l = \frac{r_2}{r_0}, \quad \beta = \left(\frac{p_0}{\rho_0} \right)^{1/2} \frac{1}{a_1}.$$

If, from theoretical considerations or from experiment, the dependence $W(l, \theta)$ is known to us, then relation (6) may be regarded as an equation for determining $l(\tau, \theta)$. Equation (6) is a nonlinear first-order partial differential equation. If at some time $t = t_0$ ($\tau = \tau_0$) the dependence

$$l = l_0(\theta), \quad (7)$$

is prescribed, then, solving the Cauchy problem for equation (6) with the initial condition (7), we find the law of variation $l(\tau, \theta)$.

2. Let us consider the question of the theoretical determination of the wave-front parameters from data on the strong stage of an explosion and the asymptotic laws of attenuation of shock waves at large distances [2, 3]. We shall regard the explosion as point-like, and the gas as perfect ($\gamma = 1.4$). As is known [6], in this case from (5) one can find $p_2(q), \rho_2(q), v_2(q)$, where $q = a_1^2/D^2$. Let

$$\Omega_1 = \Omega_2 = \exp(-z/H), \quad (8)$$

i.e., the case of an isothermal atmosphere is considered.

For small values of z/H we have

$$\Omega_1 = 1 - \frac{r}{H} \cos \theta. \quad (9)$$

For a strong explosion, the problem of a point explosion with allowance for the variation of density according to (1), (8), (9) was solved in [4, 5] by the linearization method. In accordance with the results of these works, the law of variation of the shock ...

the wave is given by the formula

$$l = \alpha^{-1/5} \tau^{2/5} [1 + \chi \alpha^{-1/5} \tau^{2/5}], \quad \chi = \frac{0.1605}{h} \cos \theta, \quad \alpha = 0.851, \quad (10)$$

and the dependence $v_2(r_2, \theta)$ is as follows:

$$v_{2\lambda} = \frac{1}{3} \sqrt{\frac{E_0}{\alpha \rho_0}} r_2^{-3/2} (1 + \chi l)^{3/2} (1 + 2\chi l). \quad (11)$$

It follows from the preceding that formula (11) gives the dependence $v_2(r_2, \theta)$ for small values of r in an explosion in an atmosphere with a density-variation law corresponding to formula (8).

As follows from the results of [2], for large distances from the explosion site the asymptotic formula is valid

$$v_{2ac} = \frac{c_1}{\rho_0 a_1} \frac{e^{ml}}{r_2} \left(\int_{r^*}^{r_2} \exp\left(\frac{r \cos \theta}{2H}\right) \frac{dr}{r} \right)^{-1/2},$$

$$m = \frac{\cos \theta}{2h}, \quad (12)$$

where c_1 and r^* are certain constants depending on the shape of the wave.

For an approximate determination of the dependence $v_2(r_2, \theta)$ over all distance ranges, we shall assume that up to some $r_*(\theta)$ formula (11) is valid, while for $r > r_*$ formula (12) takes its place. The quantities c_1 , r^* , and r_* will be chosen from the conditions for matching formulas (11), (12) at $r_2 = r_*$. In the dimensionless variables, (11), (12) take the form

$$V_\lambda = \omega l^{-3/2} (1 + \chi l)^{3/2} (1 + 2\chi l) \quad (V = v_2/a_1); \quad (13)$$

$$V_{ac} = \sigma \omega e^{ml} l^{-1} I^{-1/2} \quad \left(\omega = \frac{1}{3\sqrt{\gamma\alpha}} \right),$$

Fig. 1

Figure 1: Fig. 1

$$I = \int_{l^*}^l e^{m\xi} \frac{d\xi}{\xi} = \text{Ei}(ml) - \text{Ei}(ml^*), \quad (14)$$

Fig. 1

where $\text{Ei}(x)$ is the exponential integral function; $l^* = r^*/r^0$. We shall choose the quantity $l_*(\theta)$ from the condition that the parameter q is close to unity. We shall assume that the transition to the asymptotic formulas takes place at $q = q_*$. Since $V = 2(1 - q)/(\gamma + 1)\sqrt{q}$, the known q readily gives V , and $l_*(\theta)$ is found from formula (13).

By analogy with the spherical case [6, 7], the quantities σ and l^* will be found from the condition of matching formulas (13) and (14) and their derivatives with respect to l at $l = l_*$. To determine l^* and σ , we find the formulas:

$$\sigma = l_*^{-1/2} e^{-ml_*} I_*^{1/2} (1 + 2\chi l_*) (1 + \chi l_*)^{3/2},$$

$$I_* = \frac{1}{2} e^{ml_*} \left[ml_* + \frac{1}{2} \frac{3\chi l_*}{2(1 + \chi l_*)} - \frac{2\chi l_*}{1 + 2\chi l_*} \right],$$

where

$$I_* = \text{Ei}(ml_*) - \text{Ei}(ml^*).$$

In Fig. 1 the results are given of calculating the excess pressure $\Delta p_2/p_1$ by the method indicated above for the case $q_* = 0.9$, $h = 8$.

Since the velocity of the shock wave as a function of l, θ can be found from conditions (5), equation (6) can be used to determine $l(\tau, \theta)$. As initial data one should take the dependence $l(\tau, \theta)$ determined by equation (10) for $\tau \ll 1$. By way of example, a calculation was carried out on the “Strela” computer of the Computing Center of Moscow State University for the case $h = 8$, $l \leq 3$. The initial data were specified at $\tau_0 = 0.004$.

The Cauchy problem for equation (6) was solved by the method of characteristics. The results of the calculation are presented in Table 1. This table gives the polar

Table 1

τ	1	2	3	4	5	6	7	8	9	10	11
$\tau = \theta$ 0, 004	0	0,34910,69811,04721,39631,57081,74532,09442,44352,79253,1416									
$\tau = l$ 0, 004	0,11370,11370,11360,11360,11350,11350,11340,11330,11330,11320,1132										
$\tau = \theta$ 0, 154	0	0,35170,70311,05381,40381,57841,75282,10102,44832,79513,1416									
$\tau = l$ 0, 154	0,55410,55390,55310,55190,55050,54980,54900,54760,54650,54580,5456										
$\tau = \theta$ 0, 304	0	0,35280,70511,05651,40681,58151,75592,10362,45032,79613,1416									
$\tau = l$ 0, 304	0,80400,80350,80230,80040,79810,79680,79560,79340,79160,79040,7900										
$\tau = \theta$ 0, 504	0	0,35370,70691,05901,40961,58431,75862,10602,45202,79713,1416									
$\tau = l$ 0, 504	1,09661,09591,09411,09141,08811,08631,08461,08141,07881,07721,0766										
$\tau = \theta$ 0, 704	0	0,35450,70831,06081,41161,58641,76062,10782,45332,79783,1416									
$\tau = l$ 0, 704	1,37041,36961,36731,36381,35961,35741,35531,35121,34801,34591,3452										
$\tau = \theta$ 0, 904	0	0,35510,70941,06221,41331,58801,76222,10912,45432,79833,1416									
$\tau = l$ 0, 904	1,63471,63371,63101,62681,62181,61921,61661,61191,60801,60561,6047										
$\tau = \theta$ 1, 104	0	0,35550,71031,06341,41461,58931,76352,11032,45522,79873,1416									
$\tau = l$ 1, 104	1,89321,89211,88891,88411,87841,87541,87241,86701,86261,85981,8588										
$\tau = \theta$ 1, 304	0	0,35590,71101,06451,41571,59051,76472,11132,45592,79913,1416									
$\tau = l$ 1, 304	2,14782,14702,14302,13762,13122,12782,12452,11842,11352,11042,1093										
$\tau = \theta$ 1, 504	0	0,35630,71171,06531,41671,59151,76562,11212,45652,79943,1416									
$\tau = l$ 1, 504	2,39972,39832,39442,38842,38132,37762,37402,36722,36192,35852,3557										
$\tau = \theta$ 1, 704	0	0,35660,71231,06611,41761,59241,76652,11282,45712,79973,1416									
$\tau = l$ 1, 704	2,64962,64802,64372,63722,62942,62542,62142,61422,60842,60472,6034										
$\tau = \theta$ 1, 904	0	0,35690,71281,06681,41841,59321,76722,11352,45752,79103,1416									

$\tau = l$ 2,89782,89622,89152,88442,87612,87172,86752,85962,85342,84942,8481
1,904

coordinates of 11 points of the shock wave at various instants of time; the angles θ are given in radians. The calculations show that for $h = 8$ and $l < 3$ the shape of the shock wave differs little from a sphere.

The indicated method for determining the parameters of the blast wave can also be applied to the problem of an explosion in a nonisothermal atmosphere.

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Note: Figure translations are in progress. See original paper for figures.

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