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Abstract

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SCATTERING OF SOUND BY A CYLINDRICAL SHELL IN A MOVING MEDIUM

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Consider the scattering of a plane sound wave $p_i = \exp[ik_r \cos\varphi\, r + ik_{xx}]$ by a thin finite cylindrical shell.* It is assumed that the shell, whose axis coincides with the x-axis of the cylindrical coordinate system r, φ, x, is hinged at the points $x = 0, d$ into a cylindrical absolutely rigid screen, and that the vibrations of the shell are described by the system of equations ([1]).

Using Green' s formula, taking into account the radiation condition and the boundary condition

$$\left.\frac{\partial p}{\partial r}\right|_{r=a} = i\rho\omega\left(1 + iM\frac{1}{k}\frac{\partial}{\partial x}\right)^2 w(a,\varphi,x), \qquad 0 \leqslant x \leqslant d,$$

$$\left.\frac{\partial p}{\partial r}\right|_{r=a} = 0, \qquad -\infty < x < 0, \quad d < x < +\infty,$$

the solution of the equation

$$\left[\Delta + k^2\left(1 + iM\frac{1}{k}\frac{\partial}{\partial x}\right)^2\right] p(r,\varphi,x) = 0$$

will be represented in the form

$$p(r,\varphi,x) = p_i(r,\varphi,x) + p_r(r,\varphi,x) -$$

$$-i\rho\omega \iint_s \left[\left(1 + iM\frac{1}{k}\frac{\partial}{\partial x_1}\right) w(a,\varphi_1,x_1)\right] \widetilde{G}(a,r,\varphi_1-\varphi,x_1-x)\, ds_1. \qquad (1)$$

Here s is the surface of the shell; $p_r(r,\varphi,x)$ is the known part of the solution describing the scattering field of an absolutely rigid infinite cylinder; and $\widetilde{G}(a,r,\varphi_1-\varphi,x_1-x)$, $r > r_1 = a$, is the Green' s function, which is the solution of the equation

$$\left[\Delta_1 + k^2\left(1 - iM\frac{1}{k}\frac{\partial}{\partial x_1}\right)^2\right]\widetilde{G}(r_1, r, \varphi_1 - \varphi, x_1 - x) =$$

$$= -\frac{\delta(r_1 - r)}{r_1}\,\delta(\varphi_1 - \varphi)\,\delta(x_1 - x)$$

for $\operatorname{Im} k > 0$, bounded everywhere except the point (r, φ, x), and satisfying on the surface of the cylinder the condition

$$\left.\frac{\partial\widetilde{G}}{\partial n}\right|_s = 0,$$

$$\widetilde{G}(a, r, \varphi_1 - \varphi, x_1 - x) = \frac{1}{4\pi^2 a}\int_{-\infty}^{+\infty}\sum_{m=0}^{\infty}\frac{\varepsilon_m}{\mu H_m^{(1)\prime}(\mu a)}H_m^{(1)}(\mu r)\times$$

$$\times\cos m(\varphi_1 - \varphi)\exp[-i\xi(x_1 - x)]\,d\xi, \qquad r > a, \tag{2}$$

$$\mu = \sqrt{k^2 - (1 - M^2)\xi^2 - 2kM\xi}.$$

By substituting the solution (1) into the system of equations for the vibrations of the shell ([1]), the problem is reduced to solving a system of integro-differential equations—

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* The factor $\exp(-i\omega t)$ is omitted everywhere.

relative to $w(a, \varphi, x)$

$$\omega^2\rho_1 u + E_1\left[\frac{\partial^2 u}{\partial x^2} + \frac{1}{2a}(1-\nu)\frac{\partial^2 u}{\partial\varphi^2} + \frac{1}{2a}(1+\nu)\frac{\partial^2 v}{\partial\varphi\,\partial x} + \frac{\nu}{a}\frac{\partial w}{\partial x}\right] = \frac{i\omega}{2}\frac{\nu}{1-\nu}\frac{\partial p(a,\varphi,x)}{\partial x}; \tag{3}$$

$$\begin{gathered}\omega^2\rho_1 v + E_1\left[\frac{1}{a^2}\frac{\partial^2 v}{\partial\varphi^2} + \frac{1}{2}(1-\nu)\frac{\partial^2 v}{\partial x^2} + \frac{1}{2a}(1+\nu)\frac{\partial^2 u}{\partial\varphi\,\partial x} + \frac{1}{a^2}\frac{\partial w}{\partial\varphi}\right]\\ +\frac{1}{8}h^2E_1\frac{\nu}{1-\nu}\left(\frac{1}{a^4}\frac{\partial^3 w}{\partial\varphi^3} + \frac{1}{a^4}\frac{\partial w}{\partial\varphi}\right) = \frac{i\omega}{2a}\frac{\nu}{1-\nu}\frac{\partial p(a,\varphi,x)}{\partial\varphi};\end{gathered} \tag{4}$$

$$\omega^2\rho_1 w - E_1\left(\frac{\nu}{a}\frac{\partial u}{\partial x} + \frac{1}{a^2}\frac{\partial v}{\partial \varphi} + \frac{w}{a^2}\right) - \frac{h^2}{24}\frac{E_1}{1-\nu}\left[2(1-\nu)\left(\frac{\partial^4 w}{\partial x^4} + \frac{2}{a^2}\frac{\partial^4 w}{\partial x^2\partial\varphi^2} + \frac{1}{a^4}\frac{\partial^4 w}{\partial\varphi^4}\right)\right.$$
$$\left. +(4-\nu)\frac{1}{a^4}\frac{\partial^2 w}{\partial\varphi^2} + (2+\nu)\frac{w}{a^4}\right] = i\omega\left[\frac{1}{h} + \frac{1-2\nu}{2(1-\nu)a}\right]p(a,\varphi,x), \tag{5}$$

where

$$p(a,\varphi,x) = p_i(a,\varphi,x) + p_r(a,\varphi,x)$$
$$-\frac{i\omega\rho}{4\pi^2}\int_0^d\int_0^{2\pi}\int_{-\infty}^{+\infty}\sum_{m=0}^{\infty}\frac{\varepsilon_m H_m^{(1)}(\mu a)}{\mu H_m'^{(1)}(\mu a)}\cos m(\varphi_1-\varphi)\left[\left(1+M\frac{1}{k}\frac{\partial}{\partial x_1}\right)w(a,\varphi_1,x_1)\right]$$
$$\times \exp[-i\xi(x_1-x)]\,d\xi\,d\varphi_1\,dx_1.$$

After determining $w(a,\varphi,x)$ from equations (3)–(5) and substituting $w(a,\varphi,x)$ into (1), we obtain the desired solution $p(r,\varphi,x)$.

In expressions (1)–(5): p is the sound pressure; c is the speed of sound in the medium; M is the Mach number; $k=\omega/c$; ρ is the density of the medium; a is the radius; h is the shell thickness; $E_1 = E/(1-\nu^2)$; E is Young' s modulus; ν is Poisson's ratio; ρ_1 is the density of the shell material; n is the normal to the shell surface; u,v,w are the displacement velocities of the shell surface in the axial, circumferential, and radial directions; $H_m^{(1)}(\alpha)$ is the Hankel function of the first kind, and the prime denotes differentiation with respect to the argument; $\varepsilon_0=1,\ \varepsilon_m=2,\ldots;\ m=1,2,\ldots.$

We shall seek the solution of the system of equations (3)–(5) by expanding u,v,w and p_i+p_r in series in eigenfunctions satisfying the homogeneous differential equations of shell vibration and the self-adjoint boundary conditions at the points $x=0,d$:

$$u(a,\varphi,x) = \sum_{m,n}^{\infty} c_{mn}\cos m\varphi\sin\frac{\pi n x}{d};\qquad v(a,\varphi,x) = \sum_{m,n}^{\infty} b_{mn}\sin m\varphi\sin\frac{\pi n x}{d} \tag{6}$$

$$w(a,\varphi,x) = \sum_{m,n}^{\infty} a_{mn}\cos m\varphi\sin\frac{\pi n x}{d};\qquad p_i+p_r|_{r=a} = \sum_{m,n}^{\infty} G_{mn}\cos m\varphi\sin\frac{\pi n x}{d}.$$

Substituting (6) into (3)–(5), we obtain for a_{mn} the infinite system

$$a_{mn}Z_{mn} = G_{mn} - \sum_{l}^{\infty} Z_{mnml}a_{ml}. \quad (7)$$

We seek the solution of (7) by the method of successive approximations. Restricting ourselves, for simplicity, to the first approximation, substituting the a_{mn} found in (6), then into (1), and using the saddle-point method in integration, we obtain

$$p(r,\varphi,x) \approx p_i(r,\varphi,x) + p_r(r,\varphi,x) - i\frac{\rho c d(1-M^2)}{2\pi^2 k_r a R\cos\psi}\times \\ \times \exp\left[i\frac{kR}{1-M^2}(1-M\sin\psi)\right]\sum_{m,n}^{\infty}\frac{\varepsilon_m^2\cos m\varphi}{[Z_{mn}+Z_{mnmn}]\,H_m^{\prime(1)}(k_r a)}\times \\ \times\frac{1}{H_m^{\prime(1)}(k_r' a)}F^{(-)}(\theta,n)\left\{\left(1+M^2\frac{\pi^2 n^2}{k^2 d^2}\right)F^{(-)}(\psi,n) - 2M\frac{\pi n}{kd}F^{(+)}(\Psi,n)\right\}, \quad (8)$$

where

$$F^{(-)}(\theta,n) = \frac{\exp[i(k_x d-\pi n)]-1}{k_{xd}-\pi n} - \frac{\exp[i(k_{xd}+\pi n)]-1}{k_{xd}+\pi n};$$

$$F^{(+)}_{(-)}(\psi,n) = \frac{\exp[-i(k'_{xd}-\pi n)]-1}{k'_{xd}-\pi n} \pm \frac{\exp[-i(k'_{xd}+\pi n)]-1}{k'_{xd}+\pi n};$$

$$k_x = \frac{k}{(1-M^2)}(\sin\theta - M); \qquad k_r = \frac{k}{\sqrt{1-M^2}}\cos\theta;$$

$$k'_x = \frac{k}{(1-M^2)}(\sin\psi - M); \qquad k'_r = \frac{k}{\sqrt{1-M^2}}\cos\psi;$$

$R = \sqrt{x^2+(1-M^2)r^2}$; θ is the angle of incidence, ψ is the angle of scattering,

$$Z_{mnmn} = Z_{mnml}\big|_{l=n};$$

$$Z_{mnml} = \frac{i\rho\omega}{2\pi d}\int_0^d\int_0^d\int_{-\infty}^{+\infty}\varepsilon_m\frac{H_m^{(1)}(\mu a)}{\mu H_m^{(1)'}(\mu a)}\left[\left(1+iM\frac{1}{k}\frac{\partial}{\partial x_1}\right)^2\sin\frac{\pi n x_1}{d}\right]\sin\frac{\pi l x}{d}\exp[-i\xi(x_1-x)]\,d\xi\,dx\,dx_1;$$

$$Z_{mn} = i\frac{E_1 h}{\omega a^2}\frac{D}{D_1};$$

$$D_1 = \begin{vmatrix} -\frac{\pi n h}{2d}\frac{\nu}{1-\nu} & \frac{1+\nu}{2}m\frac{\pi n a}{d} & \frac{\rho_1\omega^2 a^2}{E_1} - \frac{\pi^2 a^2 n^2}{d^2} - \frac{1-\nu}{2}m^2 \\ \frac{mh}{2a}\frac{\nu}{1-\nu} & \frac{\rho_1\omega^2 a^2}{E_1} - m^2 + \frac{1-\nu}{2}\frac{\pi^2 a^2 n^2}{d^2} & \frac{1+\nu}{2}m\frac{\pi n a}{d} \\ -\left[1+\frac{1-2\nu}{2(1-\nu)}\right]\frac{h}{a} & -m & \nu\frac{\pi n a}{d} \end{vmatrix};$$

$$D = \begin{vmatrix} \nu\frac{\pi n}{d}a & \frac{1+\ldots}{2} \\ -m + \frac{1}{8}\frac{h^2}{a^2}\frac{\nu}{1-\nu}m(m^2-1) & \frac{\rho_1\omega^2 a^2}{E_1} - m\ldots \\ \frac{\rho_1\omega^2 a^2}{E_1} - 1 - \frac{h^2 a^2}{24}\frac{1}{1-\nu}\left[2(1-\nu)\times\left(\frac{\pi^4 n^4}{a^4} + 2\frac{m^2}{d^2}\frac{\pi^2 n^2}{d^2} + \frac{m^4}{d^4}\right) - (4-\nu)\frac{m^2}{d^4} + \frac{2+\nu}{d^4}\right] & \end{vmatrix}$$

Z_{mnmn}, Z_{mn} are, respectively, the radiation impedance and the mechanical impedance of the shell undergoing oscillations of mode number mn. If the frequency of the incident wave coincides with one of the natural frequencies of the shell in a moving medium, $\mathrm{Im}(Z_{mn} + Z_{mnmn}) = 0$, considerable scattering is observed not only in the specular direction ($\theta = \varphi$), but also in the direction opposite to that of the incident wave. However, unlike the stationary medium, where considerable scattering is observed in the direction exactly coinciding with the direction opposite to the incident wave (nonspecular reflection), in the case under consideration the direction of nonspecular reflection does not coincide with the direction opposite to the incident wave.

Let us note in conclusion that expression (8), for $M = 0$, passes into the analogous solution for a stationary medium ([2]).

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REFERENCES

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[2] L. Lyamshev, DAN, **115**, 271 (1957).

Note: Figure translations are in progress. See original paper for figures.