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Abstract

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ON THE THEORY OF IGNITION OF CONDENSED SYSTEMS BY AN INCANDESCENT SURFACE

(Presented by Academician Ya. B. Zel' dovich on 9 III 1963)

A theoretical investigation of the ignition of a gas by a hot wall under specified heat-removal conditions was carried out by Ya. B. Zel' dovich (1). In his work, as a result of solving the problem, the minimum amount of heat removed was found for which ignition at the hot wall still does not occur (a stationary regime is possible), and an ignition criterion was also obtained: at the ignition limit the heat flux at the igniting surface becomes zero. In the present note we consider the problem of ignition of nonvolatile condensed systems in whose volume exothermic chemical reactions proceed in aggregate.

Adopting a one-dimensional model of ignition, we write the heat-conduction equation with allowance for chemical sources of heat

$$\frac{\partial\theta}{\partial\tau}=\frac{\partial^2\theta}{\partial\xi^2}+e^{\theta/(1+\gamma\theta)}, \tag{1}$$

which does not take into account consumption of the k-substance during the ignition time*.

Let

$$\theta=\frac{E}{RT_s^2}(T-T_s), \qquad \tau=\frac{t}{t_a}, \qquad \xi=\frac{x}{x_a}$$

denote, respectively, the dimensionless temperature (2), time, and coordinate;

$$\gamma=\frac{RT_s}{E};$$

E is the activation energy; T is the temperature;

$$t_a=\frac{RT_s^2}{Qk_0}\,\frac{c}{Qk_0}\,e^{E/RT_s}$$

is the time of the adiabatic induction period; $x_a = \sqrt{at_a}$ is of the order of the width of the heating zone of the k-substance over the adiabatic induction period; c is the specific heat capacity; k_0 is the pre-exponential factor; Q is the thermal effect of the processes in the k-phase; a is the thermal diffusivity. Indices: s denotes the surface of the k-substance, 0 the initial instant.

We assume that at the initial instant a semi-infinite rod of the k-substance is uniformly heated to a temperature $\theta_0 < 0$; at the same instant the rod is brought into contact with a hot body having temperature $\theta_s = 0$. We assume that ideal heat exchange occurs between the surface of the body and the k-substance ($\xi = 0$), so that the temperature of the rod surface during some fixed time τ_* is equal to the temperature of the heater. After the time τ_* has elapsed, the rod is brought into contact with an adiabatic surface ($\partial\theta/\partial\xi = 0,\ \xi = 0$).

Let us consider the qualitative picture of the development of heating of the rod. Here two possibilities present themselves. Either the heat supplied by the heater and released as a result of reactions is dissipated in the bulk of the k-substance, the temperature of the rod equalizes, and after a certain time all the k-substance reacts (slow thermal decomposition); or, if by the instant τ_* the rate of heat release in the layer adjacent to the adiabatic surface is sufficiently large, then after a certain interval of time ignition will occur.

* It is assumed that consumption during the ignition time is small.

The problem consists in finding that critical time τ_* at which ignition occurs, and in determining the amount of heat supplied by the heater:

$$Q_{S_*} = -\int_0^{\tau_*} \left.\frac{\partial\theta}{\partial\xi}\right|_{\xi=0} d\tau,$$

and the amount of heat arising in the k-phase:

$$Q_{R_*} = \int_0^{\tau_*} d\tau \int_1^{\infty} e^{\theta/(1+\gamma\theta)}\, d\xi$$

up to the moment of ignition.

The mathematical formulation of the problem is as follows.

Initial condition: $\tau = 0,\ 0 \leqslant \xi < \infty;\ \theta = \theta_0 < 0.$

Boundary conditions:

a) the "hot" surface of the rod

Fig. 1 and Fig. 2

Figure 1: Fig. 1 and Fig. 2

$$\xi = 0 \begin{cases} \theta = 0 & \text{for } 0 < \tau \leqslant \tau_*, \\ \dfrac{\partial \theta}{\partial \xi} = 0 & \text{for } \tau > \tau_*; \end{cases} \tag{2}$$

b) $\xi \to \infty$:

$$\frac{\partial \theta}{\partial \xi} = 0 \quad \text{for } \tau > 0. \tag{3}$$

The sought solution of equation (1) in the most general case has the form $\theta = \theta(\xi, \tau, \tau_*, \theta_0, \gamma)$, i.e., it depends on three parameters. The course of the dimensionless curve will be determined by specific relations among these parameters.

Fig. 1. Character of the change in the temperature of the surface of the condensed substance with time. 1–above the ignition limit, 2–below the ignition limit ($\theta_0 = -10;\ \gamma = 0$)

Fig. 2. Dependence of the "critical" time τ_* on the dimensionless temperature θ_0 for $\gamma = 0$

In particular, a sharp change in the form of this curve should occur for a certain functional relation between τ_*, θ_0, γ, and consequently the critical condition for ignition will be

$$\tau_* = \tau_*(\theta_0, \gamma). \tag{4}$$

The problem of finding the dependence (4) was solved numerically by the grid method. The critical time τ_* was determined by a trial method, i.e., for each selected τ the character of the change in the dimensionless temperature $\theta(0, \tau)$ after the change of boundary condition was investigated. Ignition or extinction after the change of boundary condition was expressed quite sharply (Fig. 1). In Fig. 2

the calculated dependence $\tau_*(\theta_0)$ for $\gamma = 0$ is presented. As is seen from this figure, the critical time increases as the initial dimensionless temperature decreases.

Figure 3 gives the dependences $Q_{s*}(\theta_0)$ and $Q_{R*}(\theta_0)$. The character of these dependences coincides with the character of the change in the critical time.

The calculation showed that the dependence of $\tau_*, \theta_{s*}, Q_{R*}$ on γ is not significant, as is characterized by Table 1 for $\theta_0 = -15$.

Fig. 3. Dependences $Q_{s*}(\theta_0)$ (1) and $Q_{R*}(\theta_0)$ (2) for $\gamma = 0$

Figure 2: Fig. 3. Dependences $Q_{s*}(\theta_0)$ (1) and $Q_{R*}(\theta_0)$ (2) for $\gamma = 0$

Table 1

γ	τ_*	Q_{s*}	Q_{R*}
0	52	100	41
0.05	56	105	42

For practical calculations, one may recommend semiempirical expressions obtained by processing the results of the calculation:

$$\tau_* = 0.21\theta_0^2, \qquad \theta_{s*} = 0.44\theta_0^2, \qquad Q_{R*} = 0.17\theta_0^2.$$

Let us find the ignition conditions when the heat reserve in the heated layer corresponds to the heat reserve during combustion under stationary conditions. Obviously $Q'_{s\,\mathrm{st}} = Q'_{s*} + Q'_{R*}$ (the prime means that all Q' s are dimensional). According to O. I. Leipunskii ([3]), the heat reserve in the k-phase of a fuel burning steadily is determined by the expression

$$Q'_{s\,\mathrm{st}} = \frac{\lambda}{u}(T_k - T_0),$$

where u is the steady combustion velocity. Thus we find

$$t_* = 0.56 t_n \left(\frac{T_k - T_0}{T_s - T_0}\right)^2, \tag{5}$$

where $t_n = a/u^2$ is the relaxation time of the heated layer ([4]),

$$T_s = \left(\frac{E}{R} - 2T_0\right)\left[\ln t_* - \ln\left(0.21\frac{E}{R}\frac{c}{Qk_0}\right)\right]^{-1}.$$

Fig. 3. Dependences $Q_{s*}(\theta_0)$ (1) and $Q_{R*}(\theta_0)$ (2) for $\gamma = 0$

In particular, it follows from (5) that if the temperature of the incandescent body is equal to the surface temperature of the burning fuel specimen, then, under optimal ignition, the time of the igniting pulse is in a constant ratio (equal to 0.56) to the relaxation time of the heated layer.

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Note: Figure translations are in progress. See original paper for figures.