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Abstract

Full Text

MATHEMATICAL PHYSICS

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MASS, POLARIZATION, AND VERTEX OPERATORS IN A MODIFIED FORMULATION OF THE PROBLEM OF A NONIDEAL BOSE-EINSTEIN SYSTEM

(Presented by Academician N. N. Bogolyubov, 18 VI 1963)

In ⁽¹⁻³⁾ a method was developed, for any fixed temperature θ , for extracting from the full dynamical operator $\Omega = H - \mu N$, where H is the Hamiltonian operator and N is the operator of the total number of particles, a certain principal part Ω_0 in the asymptotic limit of weak coupling. Although, apparently, the zero dynamical operator can be chosen in many possible ways, nevertheless the choice adopted by us is very convenient in that the perturbation theory constructed on such an Ω_0 differs from other possible ones by its well-known simplicity, since in it, in each order, the contributions from certain diagrams (with paired vertices or with first-order self-energy parts, see ⁽²⁾) are substantially compensated.

In the present note we shall study the thermodynamic perturbation theory based on Ω_0 with respect to excluding the possibility of decomposing diagrams into parts connected by one solid line or by one dotted interaction line*. The main merit of the relations obtained in the present paper is that they are completely exact.

In the perturbation theory based on Ω_0 there are a dotted interaction line, as well as normal and anomalous solid particle lines.

Three types of compact parts may occur: irreducible in the sense of decomposition into two parts connected by a solid line (hatching slanted to the left); irreducible in the sense of decomposition into two parts connected by a dotted line (hatching slanted to the right); and, finally, irreducible in both senses (double hatching).

In Fig. 1 diagrams are presented for different Green' s functions and different compact parts. In Figs. 2 and 3 the corresponding completely exact integral relations are shown, which hold between the indicated quantities. These integral relations reduce to algebraic ones if, for the temperature time τ , one performs a Fourier transformation, for example,

$$G(p; \tau) = \frac{1}{\beta} \sum_k e^{-iE_{2k}\tau} G(p; E_{2k}), \quad G(p; E_{2k}) = \int_0^\beta d\tau e^{iE_{2k}\tau} G(p; \tau),$$

where $E_{2k} = 2\pi k/\beta$ and k is an integer positive number, a negative number, or zero.

Let us introduce abbreviated notation: $G^+ = G^+(p; E_{2k})$, $G^- = G^-(p; E_{2k})$; $Q = Q(p; E_{2k})$; $A = A(p)$; $B = B(p)$; $\Xi_+ = \Xi(p; E_{2k})$; $\Xi_- = \Xi(p; -E_{2k})$; $H = H(p; E_{2k})$; $\Gamma_+ = \Gamma(p; E_{2k})$; $\Gamma_- = \Gamma(p; -E_{2k})$; $\Pi = \Pi(p; E_{2k})$.

* Here only the theory of first-order mass operators is considered, associated with the decomposition of diagrams into parts connected by one line. Also important is the theory of second-order mass operators, associated with the decomposition of diagrams into parts connected by two particle lines, which we do not consider here; see ^(4, 5).

Fig. 1. Temperature diagrams for various Green's functions and various compact parts

Visible diagram labels include:

$$\mathcal{G}_0^+(p; \tau - \tau'), \quad \mathcal{G}_0^-(p; \tau - \tau'), \quad Q(q; \tau - \tau')$$

$$\mathcal{G}_0^-(p; \tau - \tau'), \quad Q^-(q; \tau - \tau'), \quad Q^-(q; \tau' - \tau)$$

$$X(p; \tau - \tau'), \quad C(p; \tau - \tau'), \quad \Xi(p; \tau - \tau')$$

$$Y(p; \tau - \tau'), \quad D(p; \tau - \tau'), \quad H(p; \tau - \tau')$$

$$L(q; \tau - \tau'), \quad P(q; \tau - \tau'), \quad \Pi(q; \tau - \tau')$$

$$K(q; \tau - \tau'), \quad R(q; \tau - \tau'), \quad \Gamma(q; \tau - \tau')$$

$$K(q; \tau' - \tau), \quad R(q; \tau' - \tau), \quad \Gamma(q; \tau' - \tau)$$

Fig. 2. First system of exact integral relations

Fig. 3. The second system of exact integral relations

Figure 3: The second system of exact integral relations

Figure 1: Figure 3: The second system of exact integral relations

Then, according to the equations shown in Fig. 2 or in Fig. 3, we obtain

$$TG^{++} = (iE_{2k} + A - \Xi_-) \left(1 + \frac{v(p)}{V} \Pi \right) + \frac{v(p)}{V} \Gamma^2 + \frac{2\sqrt{N_0}}{V} v(p) \Gamma - \frac{N_0}{V^2} v^2(p) \Pi, \quad (1)$$

$$TG^{--} = -(B - H) \left(1 + \frac{v(p)}{V} \Pi \right) - \frac{v(p)}{V} \Gamma_+ \Gamma_- - \frac{\sqrt{N_0}}{V} v(p) (\Gamma_+ + \Gamma_-) + \frac{N_0}{V^2} v^2(p) \Pi. \quad (2)$$

$$TQ = (-iE_{2k} + A - \Xi_+) (iE_{2k} + A - \Xi_-) \Pi - (B - H)^2 \Pi + \Gamma_+^2 (-iE_{2k} + A - \Xi_+) + \Gamma_-^2 (iE_{2k} + A - \Xi_-) - 2(B - H) \Gamma_+ \Gamma_-, \quad (3)$$

where

$$\begin{aligned} T = & ((-iE_{2k} + A - \Xi_+) (iE_{2k} + A - \Xi_-) - (B - H)^2) \left(1 + \frac{v(p)}{V} \Pi \right) + \\ & + \frac{2\sqrt{N_0}}{V} v(p) (\Gamma_+ (iE_{2k} + A - \Xi_-) + \Gamma_- (-iE_{2k} + A - \Xi_+) - (\Gamma_+ + \Gamma_-) (B - H)) + \\ & + \frac{1}{V} v(p) (\Gamma_+^2 (iE_{2k} + A - \Xi_-) + \Gamma_-^2 (-iE_{2k} + A - \Xi_+) - 2\Gamma_+ \Gamma_- (B - H)) - \\ & - \frac{N_0}{V^2} v^2(p) \Pi (2A - \Xi_+ - \Xi_- - 2B + 2H) - \frac{N_0}{V^2} v^2(p) (\Gamma_+ - \Gamma_-)^2. \quad (4) \end{aligned}$$

Formulas (2)–(5) are obtained from the first system of equations in Fig. 2, if the quantities X, Y, P , and R are eliminated from it. These formulas can also be obtained from the second system of equations in Fig. 3, if the quantities C, D, L , and K are eliminated from it.

The investigation presented in the present note was carried out in the course of a general program of studying the poles, or, more correctly, the analytic structure of the energy complex plane for the density Green function ⁽³⁾. The main secular equation (8) from ⁽³⁾ is obtained from (4) if (4) is set equal to zero, taking in it $\Xi = 0, H = 0, \Gamma = 0, \Pi = Q_0(q; E)$, see (6) from ⁽³⁾*

This can be done if the interaction is regarded as not only small but also long-range (the size p_0 of the potential $v(p)$ in momentum space must also be regarded as small; more precisely, one should put $p_0 = 2m^{1/2} n_0^{1/2} v^{1/2}(0) x_0$, and in addition let the dimensionless x_0 tend to zero**).

Let us note here that Hohenberg ⁽⁶⁾ has recently investigated a small and short-range interaction (the dimensionless x_0 should be taken to infinity).

The author expresses his gratitude to Hohenberg, who sent him his dissertation before its publication.

Physico-Chemical Institute
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CITED LITERATURE

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* In ⁽³⁾ it is incorrectly stated how equation (15) was obtained. In fact, this equation had already been obtained before ⁽¹⁻³⁾ by using, in thermodynamic theory, Sawada's model-Hamiltonian method ⁽⁷⁾, somewhat modified by Bogoliubov. Later this modification was independently published in Wentzel's work ⁽⁸⁾. Equation (15) is also obtained from (8) in ⁽³⁾ (and not ⁽⁷⁾, as stated there), if one puts $A(p) = E(p) + n_0 v(p)$, $B(p) = n_0 v(p)$, $\varepsilon^2(p) = E^2(p) + 2E(p)n_0 v(p)$ in $D_0(q; E)$. In $Q_0(q; E)$ one should take $\varepsilon(p) = E(p)$, $A(p) = E(p)$, $B(p) = 0$.

** We must take $p_0 \sim v^{1/2}$ in order to avoid possible nonuniform convergence of expansions in small v for small p_0 ; see (1), (2), (3) from ⁽²⁾.

Note: Figure translations are in progress. See original paper for figures.

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