THEORY OF ELASTICITY

Corresponding Member of the Academy of Sciences of the USSR A. V. POGORELOV

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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LOSS OF STABILITY OF A SHELL OF REVOLUTION UNDER EXTERNAL PRESSURE UNIFORMLY DISTRIBUTED ALONG A PARALLEL

We consider a hollow shell of revolution subjected to the action of a normal pressure uniformly distributed along a certain parallel. The problem is to determine the load at which loss of stability of the shell occurs.

1. Experience shows that by the moment at which a shell loses stability it undergoes very considerable changes of external form. In this connection, just as in the consideration of postcritical deformations, we approximate the deformed shell by an isometric transformation of the original form. A visual idea of the character of the deformations under consideration leads us to take, as a rough approximation to the form of the deformed surface, that which is obtained by double mirror reflection of the corresponding segments (Fig. 1).

Fig. 1

We assume that the form of the middle surface of the shell is well approximated by the initial form far from the buckling region (zone A_1), and is well approximated by the form of a double mirror reflection strictly inside the buckling region (zone A_2). Finally, we assume that the zone A_{12} separating them, where very substantial deformations of the shell take place, is sufficiently narrow.

2. The parallel that conventionally separates the zones A_1 and A_2 will be denoted by γ, its radius by ρ; the angle between the plane of the parallel γ and the tangent planes of the initial surface will be denoted by α. As in the consideration of postcritical deformations of a conical shell in work (^{1}), we denote by u, v the radial and axial displacements of the points of the surface under deformation. Then the equation of the surface near the parallel in cylindrical coordinates r, z will be

$$r \simeq \rho + s\cos\alpha + u(s), \qquad z \simeq s\sin\alpha + v(s);$$

s is the arc along the meridian. Neglecting the tensile-compressive deformations of the shell along the meridian, we arrive at the following relation between the displacements u, v for small α:

$$u' + \alpha v' + {}^1\!/_2\, {v'}^2 = 0.$$

As in work (^{1}), for the deformation energy of the shell one obtains the expression

$$U = \iint \frac{E\delta^3 v''^2}{24(1-\mu^2)}\, d\sigma + \iint \frac{\delta E}{2(1-\mu^2)} \left(\frac{u}{r}\right)^2 d\sigma.$$

Here the first term takes into account bending of the shell in the plane of the meridian in the neighborhood of the parallel γ, and the second term the tension-compression along the parallels in the indicated neighborhood. This expression can be transformed to the form

$$U = \frac{\pi\rho\delta E}{1-\mu^2} \int_{-\varepsilon^*}^{\varepsilon^*} \left(\frac{\delta^2 v''^2}{12} + \frac{u^2}{\rho^2}\right) ds.$$

Such a transformation is possible because of the smallness of the neighborhood of the parallel γ that is encompassed by substantial deformations. We shall denote the width of this neighborhood by $2\varepsilon^*$.

If we pass to the dimensionless variables $\bar{u}, \bar{v}, \bar{s}$, setting

$$\bar{u} = \frac{\mu u}{\varepsilon\rho\alpha^2}, \qquad \bar{v} = \frac{v'}{\alpha}, \qquad \bar{s} = \frac{\mu s}{\rho\varepsilon}, \qquad \varepsilon^4 = \frac{\mu^4\delta^2}{12\rho^2\alpha^2},$$

then, for a small value of the parameter $\delta/\rho\alpha$, as will be assumed below, the expression for the deformation energy can be given the form

$$U = \frac{\pi E \delta^{5/2}\alpha^{5/2}\rho^{1/2}}{12^{3/4}(1-\mu^2)}\, I, \qquad I = \int_{-\infty}^{\infty} (\bar{v}^2 + \bar{u}^2)\, d\bar{s}.$$

The relation between the variables $\bar{u}, \bar{v}$ will be

$$\bar{u}' + \bar{v} + \frac{\bar{v}^2}{2} = 0.$$

3. The functions u, v corresponding to the equilibrium state of the shell are determined from the condition of stationarity of the functional

$$W = U - A,$$

where U is the deformation energy of the shell, and A is the work performed by the external load. In the case under consideration

$$A = Qv,$$

where Q is the total load acting along the parallel in the axial direction, and v is the axial displacement of the parallel γ under deformation. In dimensionless variables

$$A = \frac{Q}{12^{1/4}} \sqrt{\delta\rho\alpha}\, K, \qquad K = \int_{-\infty}^{\bar{s}^*} \bar{v}\, d\bar{s},$$

where $\bar{s}^*$ is chosen in such a way that K is maximal. We assume that along the parallel γ the deflection v is maximal.

Thus, the problem of the elastic equilibrium of the shell is reduced to the consideration of the extremum problem for the functional

$$W = \frac{\pi E \delta^{5/2} \alpha^{5/2} \rho^{1/2}}{12^{3/4}(1-\mu^2)} I - \frac{Q}{12^{1/4}} \sqrt{\delta\rho\alpha}\, K$$

with the nonholonomic constraint for the varied functions

$$\bar{u}' + \bar{v} + \frac{\bar{v}^2}{2} = 0$$

and with the boundary conditions for these functions

$$\bar{u}(-\infty) = \bar{u}(\infty) = 0, \qquad \bar{v}(-\infty) = \bar{v}(\infty) = 0.$$

To determine the critical load Q, we proceed as follows. Fixing the value of the functional K, we determine the minimum of I. In this case I, and with it also W, become definite functions of K. In the equilibrium state of the shell

$$\frac{d}{dK}(U - A) = 0,$$

i.e.

$$\frac{\pi E \delta^{5/2} \alpha^{5/2} \rho^{1/2}}{12^{3/4}(1-\mu^2)} \frac{dI}{dK} - \frac{Q}{12^{1/4}} \sqrt{\delta\rho\alpha} = 0.$$

Fig. 2

Figure 2: Fig. 2

The greatest load that can be sustained by the shell, i.e. the sought upper critical load, corresponds to the greatest value of dI/dK. By a numerical method, on which we shall not dwell, it was found that

$$\max \frac{dI}{dK} \simeq 3.$$

Thus, for the magnitude of the critical load Q_k, the formula obtained is

$$Q_k = \frac{3\pi}{\sqrt{12}} \frac{E\delta^2\alpha^2}{1-\mu^2}.$$

Recall that in this formula δ is the thickness of the shell, α is the angle at which the plane of the parallel γ intersects the surface of the shell, E is the modulus of elasticity, and μ is Poisson' s ratio. For a spherical shell of radius R, $\alpha = \rho/R$. Therefore the formula may be written as

$$Q_k = \frac{3\pi E\delta^2\rho^2}{\sqrt{12}\,(1-\mu^2)\,R^2},$$

where ρ is the radius of the parallel along which the shell is loaded, and R is the radius of the shell.

4. The result obtained for a spherical shell was verified experimentally. Copper specimens of spherical shape with radius $R = 80$ mm were loaded by a force Q acting through a ring of diameter $d = 8;\ 10;\ 12$ and 14 mm. The critical value of the load Q_k, at which loss of stability occurred, accompanied by a "snap-through," was recorded. Figure 2 presents the dependence of the critical load on the radius of the parallel $\rho = d/2$ along which the loading occurred, for three shells. The individual points give the value of the critical force Q_k obtained in the indicated experiment. We see that the theoretical dependence is in good agreement with experiment.

Fig. 2

Physico-Technical Institute of Low Temperatures
Academy of Sciences of the Ukrainian SSR

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