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**Abstract**

**Full Text**

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## **REGGE POLES AND THE BETHE–SALPETER EQUATION**

*(Presented by Academician N. N. Bogolyubov, January 4, 1963)*

In previous papers <sup>(1,2)</sup> the properties of Regge poles were studied on the basis of perturbation theory. As was shown earlier <sup>(3)</sup>, the direct extraction of information from perturbation theory requires additional analysis, which, generally speaking, is nontrivial. In finding Regge trajectories by perturbation theory the difficulty arises from the fact that contributions from cuts appear and are mixed with contributions from poles. In this connection it is of interest to investigate the structure of Regge singularities on the basis of equations of the Bethe–Salpeter type. A similar approach was discussed in papers <sup>(4)</sup>. Before proceeding to the analysis of the Bethe–Salpeter equation, we shall dwell on the consideration of the Schrödinger equation with a Yukawa-type potential.

**1. The Schrödinger equation for partial amplitudes.** As is known, the Schrödinger equation has the form

$$(k^2 + E) \psi(\mathbf{k}) + \int d^3 k' V(\mathbf{k} - \mathbf{k}') \psi(\mathbf{k}') = 0. \quad (1,1)$$

In the case under consideration  $V(\mathbf{k} - \mathbf{k}')$  has the form

$$V(\mathbf{k} - \mathbf{k}') = \int_{\mu^2}^{\infty} \frac{\tilde{V}(\nu) d\nu}{(\mathbf{k} - \mathbf{k}')^2 + \nu}. \quad (1,2)$$

Let us write equation (1,1) for partial amplitudes. For this purpose we represent  $\psi(\mathbf{k})$  in the form

$$\psi(\mathbf{k}) = \sum_{l=0}^{\infty} (-i)^l \frac{1}{k} \psi_l(k) P_l(\cos \theta_k). \quad (1,3)$$

Using the representation

$$\frac{1}{(\mathbf{k} - \mathbf{k}')^2 + \mu^2} = \frac{1}{2kk'} \sum (2l + 1) Q_l \left( \frac{k^2 + k'^2 + \mu^2}{2kk'} \right) P_l(\cos \theta_{\mathbf{k}\mathbf{k}'}), \quad (1,4)$$

and then substituting (1,3) and (1,4) into (1,1), we obtain:

$$(k^2 + E)\psi_l(k) + 2\pi \int_{\mu^2}^{\infty} d\nu \int_0^{\infty} dk' \tilde{V}(\nu) Q_l \left( \frac{k^2 + k'^2 + \nu}{2kk'} \right) \psi_l(k') = 0. \quad (1,5)$$

The partial amplitudes in  $r$ -space

$$\tilde{\psi}(\mathbf{r}) = \sum_{l=0}^{\infty} \frac{1}{r} \tilde{\psi}_l(r) P_l(\cos \theta_r) \quad (1,6)$$

are related to the partial amplitudes  $\psi_l(k)$  as follows:

$$\begin{aligned} \psi_l(k) &= \int_0^{\infty} \tilde{\psi}_l(r) J_{l+1/2}(kr) \sqrt{kr} dr, \\ \tilde{\psi}_l(r) &= \int_0^{\infty} \psi_l(k) J_{l+1/2}(kr) \sqrt{kr} dk. \end{aligned} \quad (1,7)$$

When calculating the Regge trajectory it is necessary to keep the following circumstance in mind. The Regge index is a function of the parameter  $e^2/\sqrt{E}$ , which, in the study of bound states, is not small and is of order unity. Therefore, in order to find  $\alpha(E)$  we must be able to sum all terms of the type  $(e^2/\sqrt{E})^n$ . Let us illustrate this with the example in which the potential in equation (1.5) is equal to the sum of the Coulomb and Yukawa potentials

$$\tilde{V}(\nu) = -\frac{1}{2\pi^2} [e^2\delta(\nu) + g^2\delta(\nu - \mu^2)]. \quad (1,8)$$

Equation (1.5) then becomes

$$\begin{aligned} (k^2 + E)\psi_l(k) - \frac{e^2}{\pi} \int_0^{\infty} dk' Q_l \left( \frac{k^2 + k'^2}{2kk'} \right) \psi_l(k') = \\ = \frac{g^2}{\pi} \int_0^{\infty} dk' Q_l \left( \frac{k^2 + k'^2 + \mu^2}{2kk'} \right) \psi_l(k'). \end{aligned} \quad (1,9)$$

In order to sum all terms of order  $(e^2/\sqrt{E})^n$  for the zeroth approximation in (1.9), it is necessary to take the Coulomb partial amplitudes. In this approximation the principal Regge trajectory has the form

$$\alpha_0(E) = -1 + \frac{e^2}{2\sqrt{E}}. \quad (1.10)$$

Treating the right-hand side as a perturbation, one can obtain, for the Regge trajectory of equation (1.9), in this approximation, the expression

$$\alpha = -1 + \frac{e^2}{2\sqrt{E}} + \frac{g^2}{2\sqrt{E}} \left( 1 + \frac{\mu}{2\sqrt{E}} \right)^{-e^2/\sqrt{E}}. \quad (1.11)$$

The above choice of the zeroth approximation made it possible to obtain the correct expansion in the small parameter  $g^2$ .

## 2. Bethe-Salpeter equation for partial amplitudes

As is known, the Bethe-Salpeter equation for a scalar two-particle wave function has the form

$$\psi(\varepsilon, \mathbf{k}) = -\lambda F(\varepsilon, \mathbf{k}) \int \frac{\psi(\varepsilon', \mathbf{k}') d\varepsilon' d^3k'}{(\mathbf{k} - \mathbf{k}')^2 - (\varepsilon - \varepsilon')^2 + \mu^2}, \quad (2.1)$$

where

$$F(\varepsilon, \mathbf{k}) = G(\varepsilon + E, \mathbf{k})G(\varepsilon - E, \mathbf{k}),$$

$$G^{-1}(\varepsilon + E, \mathbf{k}) = (\varepsilon + E)^2 - k^2 - m^2 + i\delta, \quad (2.2)$$

$$\lambda = \frac{4i}{(2\pi)^4} g^2.$$

As a zeroth approximation in equation (2.1), it is expedient to explicitly separate out the part of potential character. For this purpose we rewrite the equation in the form

$$\begin{aligned} \psi(\varepsilon, \mathbf{k}) = & -\lambda F(\varepsilon, \mathbf{k}) \int Q(\mathbf{k}, \mathbf{k}') \psi(\varepsilon', \mathbf{k}') d\varepsilon' d^3k' - \\ & -\lambda F(\varepsilon, \mathbf{k}) \int d\varepsilon' \left[ \frac{1}{(\mathbf{k} - \mathbf{k}')^2 - (\varepsilon - \varepsilon')^2 + \mu^2} - Q(\mathbf{k}, \mathbf{k}') \right] \psi(\varepsilon', \mathbf{k}') d^3k'. \end{aligned} \quad (2.3)$$

Integrating equation (2.3) with respect to  $\varepsilon$ , we have:

$$\begin{aligned} \psi(\mathbf{k}) = & -\lambda F(\mathbf{k}) \int Q(\mathbf{k}, \mathbf{k}') \psi(\mathbf{k}') d^3 k' - \\ & -\lambda \int d\varepsilon d\varepsilon' F(\varepsilon, \mathbf{k}) \left[ \frac{1}{(\mathbf{k} - \mathbf{k}')^2 - (\varepsilon - \varepsilon')^2 + \mu^2} - Q(\mathbf{k}, \mathbf{k}') \right] \psi(\varepsilon', \mathbf{k}') d^3 k', \end{aligned} \quad (2.4)$$

where

$$\begin{aligned} \psi(\mathbf{k}) = & \int \psi(\varepsilon, \mathbf{k}) d\varepsilon, \\ F(\mathbf{k}) = & \int F(\varepsilon, \mathbf{k}) d\varepsilon = \frac{i\pi}{2\sqrt{k^2 + m^2} (k^2 + m^2 - E^2)}. \end{aligned} \quad (2.5)$$

We determine the kernel  $Q(\mathbf{k}, \mathbf{k}')$  from the condition that the second term on the right-hand side of equation (2.4) vanish. In the lowest approximation this condition gives for the kernel  $Q(\mathbf{k}, \mathbf{k}')$  the expression

$$Q(\mathbf{k}, \mathbf{k}') = \frac{1}{F(k)F(k')} \iint d\varepsilon d\varepsilon' \frac{F(\varepsilon, \mathbf{k})F(\varepsilon', \mathbf{k}')}{(\mathbf{k} - \mathbf{k}')^2 + \mu^2 - (\varepsilon - \varepsilon')^2}. \quad (2.6)$$

In this approximation equation (2.4) is written in the form

$$(k^2 + m^2 - E^2)\psi(\mathbf{k}) = \frac{-i\pi\lambda}{2\sqrt{k^2 + m^2}} \int Q(\mathbf{k}, \mathbf{k}') \psi(\mathbf{k}') d^3 k'. \quad (2.7)$$

Passing in equation (2.7) to partial amplitudes, we obtain for  $\alpha$  an expansion of the form

$$\alpha = \alpha_0 + \frac{g^2}{4\pi m^2} \psi\left(\frac{g^2}{m\sqrt{m^2 - E^2}}\right) + \dots,$$

where

$$\alpha_0 = -1 + \frac{g^2}{4\pi m^2} \frac{m}{2\sqrt{m^2 - E^2}}.$$

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