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Abstract

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On the Structure of Layer F and the Causes of Convection in the Earth' s Core

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Hypothesis on the structure of layer F. On the basis of seismic data, Jeffreys ([1]) concluded that at the boundary of the inner core, at $r = 1250$–1390 km, there is a transition layer (layer F) in which the velocity of longitudinal waves decreases toward the center approximately proportionally to r. Gutenberg, however, believes that in the transition layer the velocity increases monotonically toward the center ([2]). Bullen ([3]), proceeding from his hypothesis according to which compressibility is determined mainly by pressure, indicated that the drop in velocity in layer F is caused by an increase in its density.

It seems natural to explain this sharp increase in density in layer F by assuming that, in addition to iron, the liquid core contains a light admixture, probably silicon, which is gradually frozen out during crystallization of the inner core under the action of pressure. A number of authors have pointed to the probable presence of silicon in the core, for example ([4]). The presence of silicon is also consistent with Ringwood' s qualitative picture of the formation of the Earth ([5]).

The structure of the transition layer can be clarified with the aid of the phase diagram of Fe–Si in the coordinates p, ξ (pressure, concentration). Its presumed form is shown in Fig. 1, by analogy with the T, ξ diagram for Fe–Si at $p = 1$ atm. ([6]). At $r = R_F = 1390$ km precisely the pressure p_F is reached, corresponding to the point f on the liquidus line. Here the substance of the liquid core with concentration ξ_0 is in equilibrium with crystals of higher concentration ξ_c (if at these pressures the compound FeSi still exists, then $\xi_c = 33.4\%$). However, crystallization cannot occur here, since crystals with concentration ξ_c are lighter than the surrounding liquid, and therefore they would immediately float up and melt. Further, as the pressure increases in layer F, the liquid lies on the liquidus curve fb, and crystallization in it can occur until the eutectic point b is reached. Here, heavy crystals with a low silicon concentration ξ_g are simultaneously formed from the liquid; these build up the inner core, and light crystals, which float up and melt.

As a result of convection and thermal conductivity, a certain temperature gradient is established in layer F. Thus Fig. 1, where the axes p and r are combined,

Fig. 1

Figure 1: Fig. 1

should be regarded not as a p, ξ diagram at $T = \text{const}$, but as such a section of the phase surfaces in which the temperature varies together with pressure in the same way as in layer F. This, however, does not affect the qualitative picture of the structure of the layer given above (provided, of course, that the temperature does not increase with depth too rapidly, so that crystallization is at all possible). We note that the same qualitative picture will also be preserved for another type of phase diagram, if only the liquidus curve has $d\rho/d\xi < 0$, so that density increases with pressure.

Jeffreys'effect (the drop in velocity in layer F) can be explained by the increase of density in the layer associated with the change in concentration along the liquidus line. The wave velocity $u = (k/\rho)^{1/2}$ changes mainly as a result of

···of the change in density ρ. Considering this change small and neglecting the curvature of the liquidus line, we obtain that the velocity changes linearly in the layer F. For a layer thickness $R_F - R_G = 0.1R_F$, the relation $u/u_F = r/R_F$ will hold if the density in the layer changes by 20%. Taking the increase in incompressibility into account will increase this value somewhat.

On the other hand, one may compare the density of the core ρ with the density of iron ρ_{Fe}, which is now determined experimentally (7). For Bullen' s models A and B (3) the ratio ρ_{Fe}/ρ ranges from 1.21 to 1.14. In its very meaning this estimate should give a larger density ratio than the preceding one, but, taking into account some uncertainty both in the layer thickness and in the core density, it may be assumed that the two estimates are in agreement. In this case, however, it is necessary to assume that in the diagram of Fig. 1 ξ_b, ξ_g are small, i.e., that the solubility of silicon in solid iron is small at high pressures. Such an assumption appears natural. As is known (8), the solubility of metals depends on the ratio of their atomic diameters d, and at $p = 1$ atm solubility ceases when this difference reaches approximately 15%. At $p = 1$ atm we have $d_{\text{Fe}}/d_{\text{Si}} = 1.08$, $\xi_g = 18.5\%$, $\xi_b = 20\%$. At high pressures the role of volume factors should greatly increase, while the difference of atomic diameters also increases somewhat; therefore one may expect ξ_g to be very small. The assumption that a small amount of nickel is present in the core also brings the two estimates closer together.

Fig. 1

Let us estimate the silicon content ξ_0 in the liquid core. At $p = 1$ atm, the ratio $\rho_{\text{Fe}}/\rho = 1.2$ corresponds to $\xi = 26\%$ (9). This value must increase somewhat because of the increase in the ratio $d_{\text{Fe}}/d_{\text{Si}}$, while the presence of nickel, on the contrary, acts to reduce the estimate of ξ_0. We note that if ξ_0 is close to 33.4%, then a small change of pressure near the point F corresponds to a large change of ξ and ρ and, consequently, to a sharp decrease in the velocity of seismic waves near R_F.

Gutenberg effect –dispersion of waves in the transition layer (2). Gutenberg found that short waves with periods $T \leq 1$ sec, passing through the transition layer, have there a greater velocity (by about 10%) than long waves $T \geq 2$ sec. According to Gutenberg, the velocity in the transition layer increases monotonically from the value corresponding to the liquid to the value corresponding to the solid core, and the dispersion effect is connected with the gradual solidification of the material.

Another explanation of the Gutenberg effect may be proposed, based on the above-adopted idea of the structure of the layer. In the layer F the state of the material corresponds to the melting point; therefore pressure and temperature oscillations in a wave must cause a phase transition. If the characteristic time of this transition is ~ 1 sec, then in long waves ($T > 2$ sec) the change of pressure occurs in an equilibrium manner, with the transition of some fraction of the liquid phase into the solid one, and the velocity is determined by the equilibrium value of the derivative $(\partial p/\partial \rho)_0$, whereas in short waves ($T < 1$ sec) equilibrium does not have time to be established, and with increasing frequency their velocity tends to the limiting value determined by the derivative $(\partial p/\partial \rho)_\infty$ for the liquid phase. According to Le Chatelier' s principle, $(\partial p/\partial \rho)_\infty > (\partial p/\partial \rho)_0$, and therefore short waves propagate faster than long ones; moreover, short waves should experience additional attenuation (10).

On the nature of convection in the Earth' s core. To maintain the adiabatic temperature gradient ∇T_a in the core, a flux is necessary···

heat $q_a = -\varkappa \nabla T_a$, amounting, for example, according to the estimate of Ballard and Hellman [11], to $2.6 \cdot 10^{-7}$ cal/cm^2 • sec.

Verhoogen [12] noted that the role of radioactive heat is small, and considered the release of heat occurring during the cooling and crystallization of the inner core, due to heat capacity and the latent heat of fusion. He showed that the cooling of the Earth may be the cause of convection. According to our hypothesis concerning the structure of layer F, it is necessary, in addition, to take into account the heat released at the expense of gravitational energy because of the freezing out of a light component during crystallization.

Let, during a time δt, the core cool by δT; then the boundary R_2 of the inner core, where the temperature is related to the pressure by the melting condition $T = T_m(p)$, shifts outward by δR_2, so that the decrease in pressure $\delta p = \rho g_2 \delta R_2$ compensates the cooling. Outside, in the liquid core, the temperature is related to the pressure adiabatically, $dT_a/dp = \alpha T/\rho c_p$, therefore

$$\delta T = \left(\frac{dT_m}{dp} - \frac{dT_a}{dp}\right)\delta p = \left(\frac{dT_m}{dp}\rho g_2 - \frac{\alpha T g_2}{c_p}\right)\delta R_2.$$

Here α is the coefficient of thermal expansion, c_p is the specific heat. The total heat release in this case is

$$\delta Q = c_{pM} 1 \delta T + (w_m + U)\delta M_2 \equiv w_{\text{eff}} \delta M_2.$$

Here $c_p \approx 0.2$ cal/g $\cdot$ deg; $M_1 = 1.9 \cdot 10^{27}$ g is the mass of the whole core; $\delta M_2 = \rho 4\pi R_2^2 \delta R_2$ is the increase in the mass of the inner core; w_m is the heat of fusion (according to [12], $w_m \approx 10^2$ cal/g); $U \approx 0.5\, g_1 R_1\, \Delta\rho/\rho$ is the release of gravitational energy (per unit mass) due to the increase in density upon crystallization by $\Delta\rho$ (the coefficient 0.5 roughly accounts for the redistribution of density); $R_1 = 3.47 \cdot 10^8$ cm is the radius of the outer core; $g_1 = 10^3$ cm/sec^2. Taking $\Delta\rho/\rho = 0.2$, we obtain $U = 3.5 \cdot 10^{10}$ erg/g $= 830$ cal/g, i.e., much larger than w_m. The contribution to w_{eff} from the first term is equal to $w_T = (c_{pM} 1/\rho 4\pi R_2^2)\, \delta T/\delta R_2$. Simon' s approximate formula [13] for iron gives $T_m = 3900$°K, $dT_m/dr = (dT_m/dp)\rho g_2 = 0.13$ deg/km. If for iron, according to [7], we take $\alpha = 10^{-5}$ deg^{-1}, then we obtain the adiabatic gradient $dT_a/dr = \alpha g_2 T/c_p = 0.18$ deg/km, which is larger than the gradient of the melting point dT_m/dr, although by the sense of the matter it should be the other way around. This once again indicates that the liquid core is not pure iron. Adopting, in the absence of a better estimate, $dT_m/dr - dT_a/dr \approx 0.5\, dT_m/dr$, we obtain $w_m \approx 10^2$ cal/g. As a result we obtain $w_{\text{eff}} = w_T + w_m + U \approx 10^3$ cal/g, with the principal role being played precisely by the release of gravitational energy.

It should be noted that the "release of heat" w_{eff} is inseparably connected with cooling, so that it would be better to speak of an approximately order-of-magnitude increase in the effective heat capacity. The rate of heat release $\delta Q/\delta t = w_{\text{eff}} \delta M_2/\delta t$ may be estimated very roughly by putting $\delta M_2/\delta t = M_2/t$, where $M_2 = 10^{26}$ is the mass of the inner core and $t = 4 \cdot 10^9$ years is the age of the Earth. This gives $\delta Q/\delta t \sim 7 \cdot 10^{11}$ cal/sec and a heat flux at the boundary of the liquid core $(\delta Q/\delta t)/4\pi R_1^2 \sim 5 \cdot 10^{-7}$ cal/cm$^2 \cdot$ sec, which is in reasonable agreement with the flux from the adiabatic temperature gradient.

The immediate cause of convection is the Archimedean force $f = -g\rho(\alpha T' + \bar{\alpha}\xi')$, where $T' = T - T_a$, $\xi' = \xi - \xi_0$ are deviations from the corresponding static values; $\bar{\alpha} = -\rho^{-1}(\partial\rho/\partial\xi)$. Let us compare the terms in f. Equating the density of release of gravitational energy, $-\bar{\alpha} g \rho \bar{D} \nabla \xi' \sim -\bar{\alpha} g \rho \bar{D} \xi'/L$, and $-\operatorname{div}(\tilde{\varkappa} \nabla T') \sim \tilde{\varkappa} T'/L^2$, where $L \lesssim R_1$ is the characteristic size; $\bar{D}$, $\tilde{\varkappa}$ are the coefficients of diffusion and thermal conductivity (turbulent), we obtain $\alpha T'/\bar{\alpha}\xi' \sim \alpha g L \rho \bar{D}/\tilde{\varkappa}$. Transfer of concentration and heat is provided by one and the same mechanism of turbulent convection, and therefore one may put $\tilde{\varkappa} \sim \rho c_p \bar{D}$. In this case $\alpha T'/\bar{\alpha}\xi' \sim \alpha g L/c_p \sim 10^{-1}$, i.e., convection in the core is not thermal; the change in density is connected mainly with the change in concentration. The freezing out of the light component during crystalliza-

increases the release of heat severalfold and, consequently, the term $\alpha T'$, but it affects convection still approximately an order of magnitude more strongly, directly creating the Archimedean force $-g\rho\bar{\alpha}\xi'$. At the lower boundary of the liquid core an excess of the light component is formed during crystallization; its rise is the principal cause of convection in the core.

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Note: Figure translations are in progress. See original paper for figures.